An introduction to modeling interferometric data.

EuroWinter School
Observing with the Very Large Telescope Interferometer

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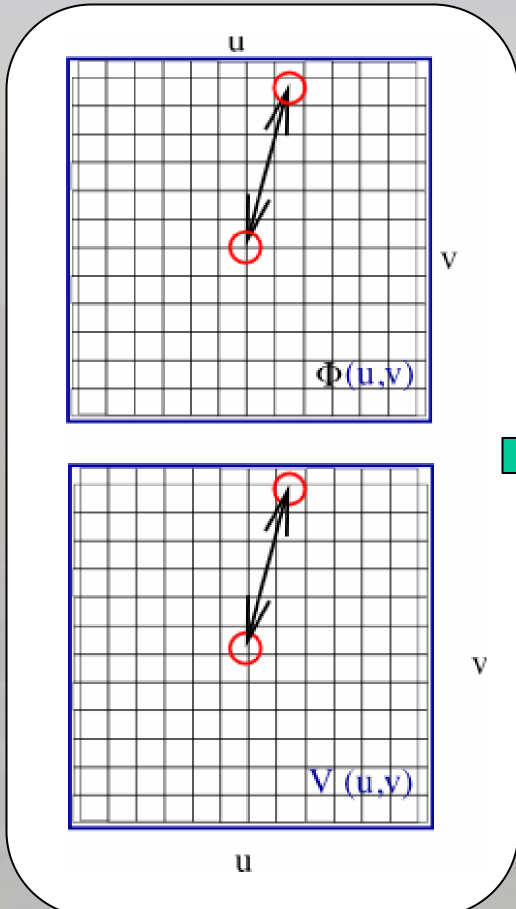
Imaging with an interferometer

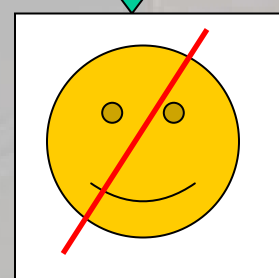
Practical application of the Van-Cittert Zernike theorem

$U = Bx/\lambda$

$V = By/\lambda$



$$I(x,y) = FT^{-1}(V(u,v))$$



Y

X

Model fitting

This talk adresses the basic issues of interpreting visibilities directly

→ **Realistic in the VLTI AMBER and MIDI contexts**

Model fitting in the visibility domain is a very attractive complement (alternative) to imaging:

- domain where measurements are made-> errors easier to recognize
- when (u,v) plane sampling is poor
- might be better to address some issues such as source variability

OUTLINE

1. Modeling visibilities: principles.
2. Some useful basic functions.
3. Standard issues.
4. Constraining the parameter space
5. Using your own model

The modeling process.

Model

Parameters α, β, γ

$$I(x, y)$$

$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y)\exp(-2\pi i(xu + yv))dxdy$$

This talk

- Perfect (u,v) sampling
- Monochromatic

Instrument

Sparse sampling $S(u,v)$

Error $\epsilon(u,v)$

$$\{..., V(u_i, v_i), ...\} i = 1..n$$

Data & Error

$$\{..., V'(u_k, v_k), ...\} k = 1..n$$

A minimization process:

$$\chi^2 = f(V(u_i, v_i), V'(u_k, v_k), \epsilon'(u,v))$$

Best α, β, γ

Ad-hoc modeling

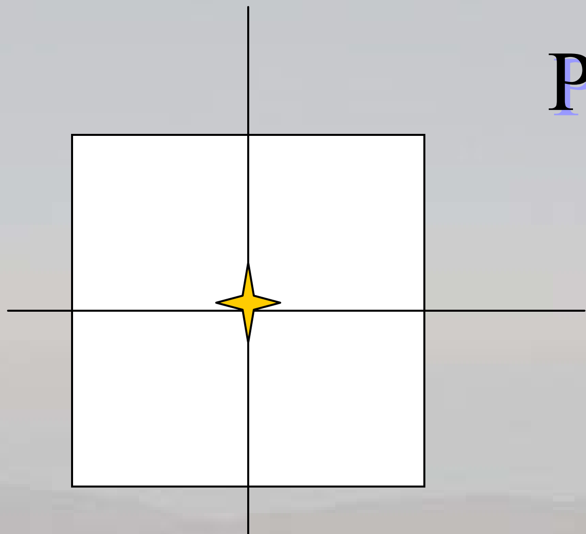
Fourier transform properties
Use of basic intensity distribution functions .

Important first step towards modeling with real physical model

Fourier transform properties:

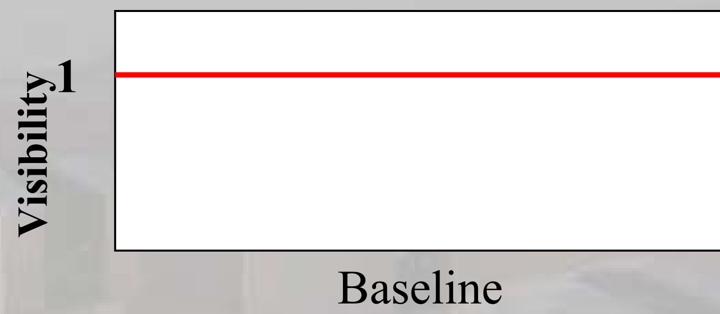
1. **Addition** $\mathrm{FT}\{f(x,y)+g(x,y)\} = F(u,v)+G(u,v)$
2. **Convolution** $\mathrm{FT}\{f(x,y)\times g(x,y)\} = F(u,v).G(u,v)$
3. **Shift theorem** $\mathrm{FT}\{f(x-x_0,y-y_0) = F(u,v)\exp[2\pi i(ux_0+vy_0)]$
4. **Similarity theorem** $\mathrm{FT}\{f(ax,by)\} = \frac{1}{|ab|}F(u/a,v/a)$

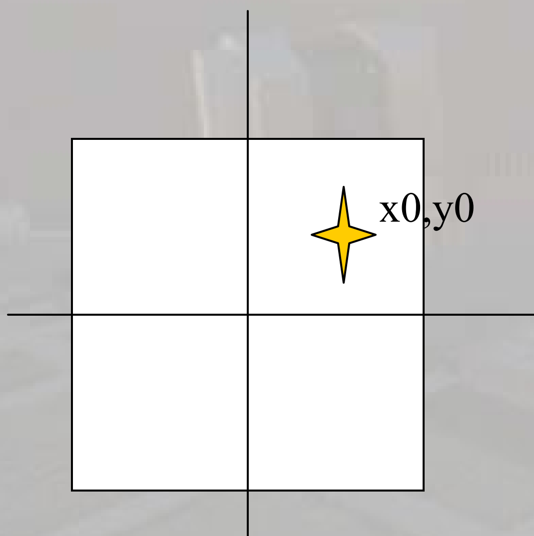
Point source function



Centered source

$$I(x,y) = \delta(x,y) \Rightarrow V(u,v) = 1$$





Off-axis source

$$I(x,y) = \delta(x - x_0)\delta(y - y_0) \Rightarrow V(u,v) = \exp[-2i\pi(x_0 u + y_0 v)]$$

Amplitude =1 , linear dependence for the phase

Gaussian brightness distribution.

Use: Estimate for angular sizes of envelopes-disks etc

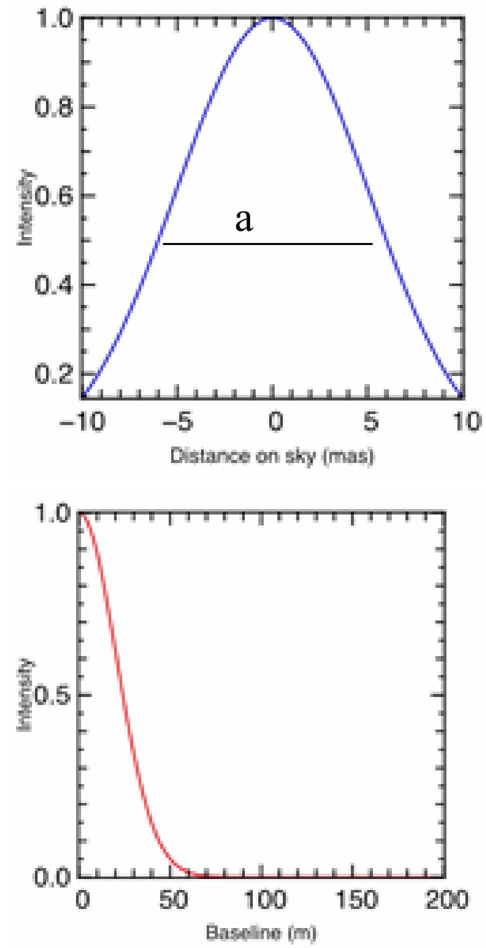
$$I(r) = \frac{I_0}{\sqrt{\pi/4\ln 2}\,a}\exp(-4\ln 2 r^2/a^2)$$

$$V(\rho) = \exp[-(\pi a \rho)^2/(4\ln 2)]$$

Where a: FWHM intensity, Io Peak intensity

with $r = \sqrt{x^2 + y^2}$

with $\rho = \sqrt{u^2 + v^2}$



Uniform disk

Use: aproximation for brightness distribution of photospheric disk.

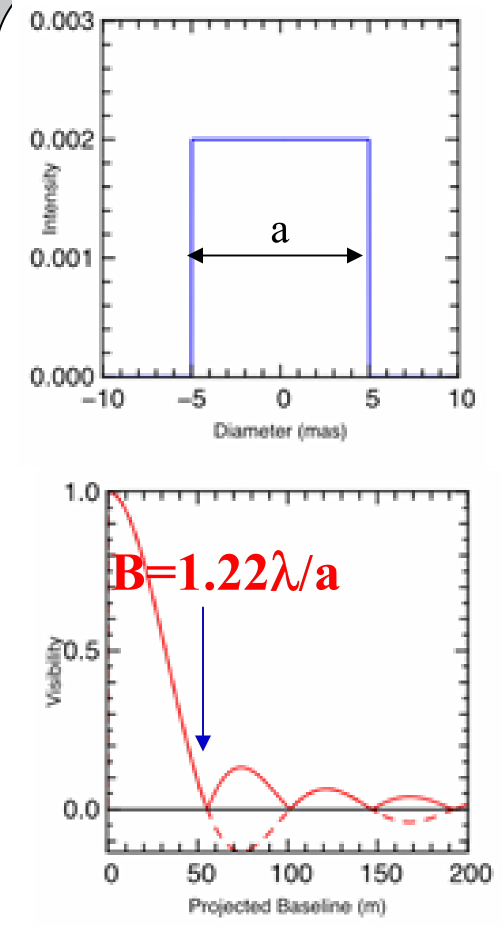
$$\begin{aligned} \mathrm{I(r)} &= 4/(\pi a^2), \mathrm{if r} = \sqrt{\mathrm{x^2 + y^2}} \leq \mathrm{a}/2 \\ \mathrm{I(r)} &= 0 \quad \mathrm{otherwise} \end{aligned}$$

$$F(\rho) = \frac{J_1(\pi a \rho)}{\pi a \rho} \mathrm{with} \rho = \sqrt{\mathrm{u^2 + v^2}}$$

a: diameter

Sophistication of the model I= f(r), limb darkening

Cf Young

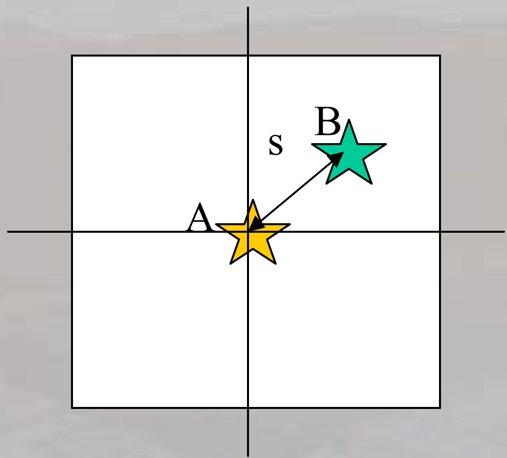


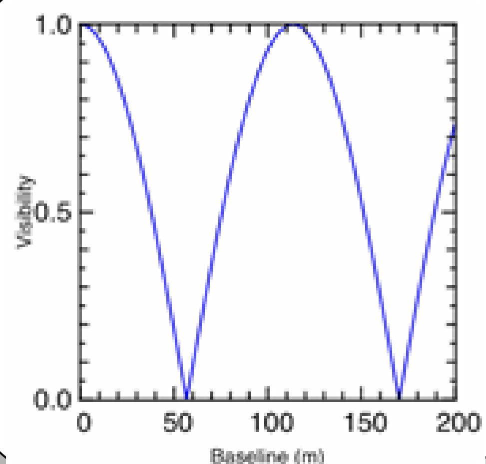
Binary

Unresolved

$$A\delta(x,y) + B\delta(x - sx, y - sy) \text{ with } s = \sqrt{sx^2 + sy^2}$$

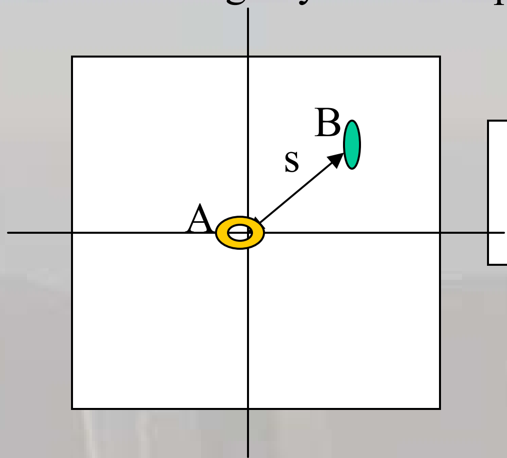
$$\begin{aligned} \mathrm{V(u,v)} &= \sqrt{\frac{1 + r_{ab}^2 + 2r_{ab}\cos 2\pi \vec{L_b}\vec{s}/\lambda}{1 + r_{ab}^2}} \\ \text{with } \mathrm{r}_{ab} &= \mathrm{A/B} \\ \text{with } \vec{L_b} &= \text{Baseline vector} \end{aligned}$$





Resolved bi-structure

Use: Describing any multicomponent structure.



$$V^2(u,v) = \frac{r_{ab}^2 * V_a^2 + V_b^2 + 2 r_{ab} |V_a||V_b| \cos(2\pi \vec{L_b}\vec{s}/\lambda)}{(1 + r_{ab}^2)}$$

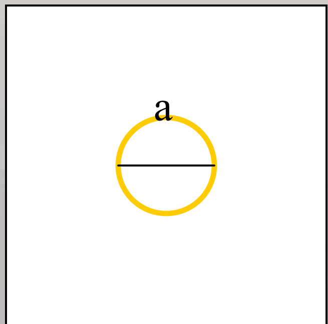
Where Va and Vb are respectively the visibility of object A and B at baseline (u,v)

Generalization:

$$V(u,v) = \frac{\sum_{i=1}^{k} F_i V(u_i, v_i)}{\sum_{i=1}^{k} F_i}$$

Unresolved ring & Ellipse

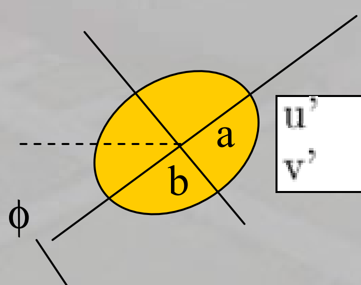
Use: allowing to describe a more complex centro-symmetric structure and compute its visibility



$$I(r) = 1/(\pi a)\delta(r - a/2)$$

$$V(\rho) = J_0(\pi a \rho)$$

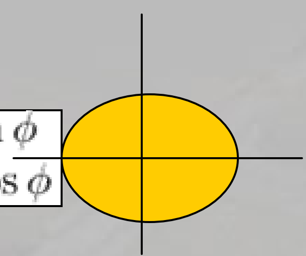
Exercize



(u,v) plane

Rotation

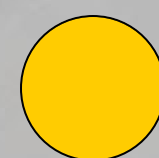
$$\begin{aligned} u' &= u\cos\phi + v\sin\phi \\ v' &= -u\sin\phi + v\cos\phi \end{aligned}$$



(u',v') plane

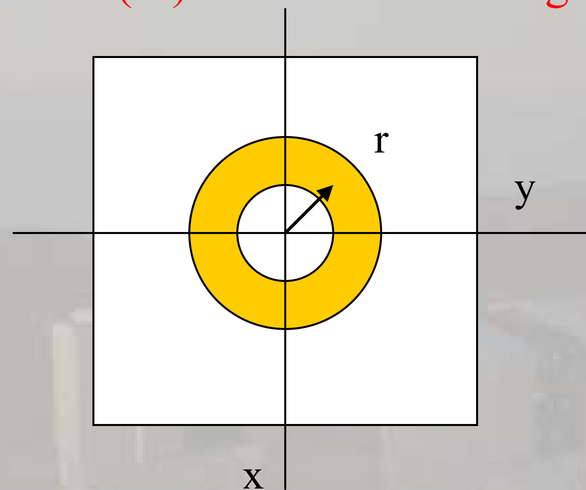
Similarity

$$\begin{aligned} x' &= x/a \\ y' &= y/b \end{aligned}$$



Circularly symmetric component

Circularly symmetric component I (r) centered at the origin of the (x,y) coordinate system.



with $r = \sqrt{x^2 + y^2}$

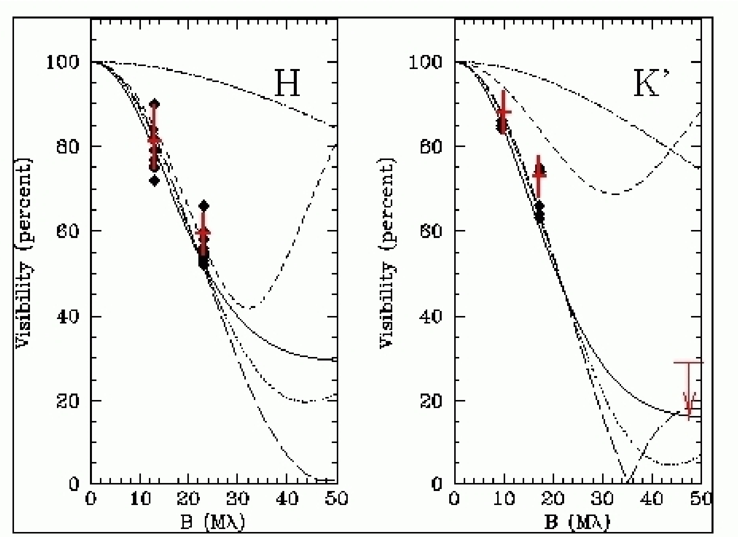
The relationship between brightness distribution and visibility is a Hankel function

$$V(\rho) = 2\pi \int_0^{\infty} I(r) J_0(2\pi r \rho) r dr$$

with $\rho = \sqrt{u^2 + v^2}$

Classical issues

- Model fitting is also a deconvolution process: sizes estimates or positional uncertainties can smaller than the canonical resolution (the “beam” size”): **super resolution**
- If the object is barely resolved the exact brightness distribution is not crucial - the dependance is quadratic for all the basic functions.



Classical issues

- An extended source contributing weakly to the total flux level will have as high visibility As a more compact source.

- Choosing the observational parameters upon their constraining force is useful: use model derivatives.

Conclusion

- ✓ Visibility study without imaging can be efficient.
- ✓ It is the natural way to understand the errors of the final result.
- ✓ Always start by describing your observations in terms of basic functions. It brings quantitative information useful for further more detailed computations.
- ✓ Use basic models in order to prepare your observation and determine

what is the more constraining configuration.