Introduction to Data Reduction

EuroWinter School

Observing with the Very Large Telescope Interferometer

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Topics

System Analysis

- The perturbed wavefront
- Diffraction effects: the pupil function
- Image spectrum: the transfer function
- The field of view
- Spatial filtering

Data processing

- The observables
- Interferometric equation
- Temporal coding
- Wavelet analysis
- Spatial coding
  - Visibility
  - Differential phase
  - Closure phase
- The AMBER case

The perturbed wavefront

- Object wavefront → $\Psi_o(x)$
- Atmospheric effects → $e^{i\phi(x,t)}$
- Perturbed wavefront → $\Psi(x,t) = \Psi_o(x)e^{i\phi(x,t)}$

The perturbed wavefront is defined by $N_s = (\frac{D}{r_0})^2$ random phases

➡ Calibration problems

Diffraction effects: the pupil function

- Fraunhofer diffraction → $A(\alpha,t) \propto \int_{Aperture} \Psi(x,t) e^{-2i\pi\alpha\frac{x}{\lambda}} dx$
- Pupil function with → $P(u)$ with $u = \frac{x}{\lambda}$

$$A(\alpha,t) \propto \int \Psi(u,t) P(u) e^{-2i\pi\alpha u} du$$

$$A(\alpha,t) \propto FT\left[\Psi(u,t)P(u)\right]$$

The diffracted field is proportional to the Fourier transform of the input field

Image spectrum: the ideal transfer function

Image plane intensity distribution → $I(\alpha) =< |A(\alpha)|^2 > \propto < |FT[\Psi_o(u)P(u)]|^2 >$

Image spatial spectrum → $I(f) \propto \int < \Psi_o^*(u)\Psi_o(u+f) > P^*(u)P(u+f)du$

Zernike-van Cittert theorem → $< \Psi_o^*(u)\Psi_o(u+f) > \propto O(f)$

Image spectrum → $I(f) = O(f).T(f)$

Ideal transfer function → $T(f) = \frac{1}{s} \int P(u)P^*(u+f)du$

The field of view (FOV)

- Theorem: the Fourier transform of a periodic function with period L is *discrete* at points separated with $\Delta f = \frac{1}{L}$
- These points are called *independant points*

- If we know the Fourier components of an image with a sampling interval $\Delta f = \frac{1}{L}$, it is then possible to reconstruct a field L

- Single telescopes have continous transfer function
  → the FOV is only limited by geometrical constraints

- Diluted telescopes have discontinous transfer function
  → the FOV is limited by the sampling of the pupil plane

⇒ Importance of positivity and *support* for image reconstruction

Spatial filtering

- Non fibered (multimode) interferometers transmit the object spectrum over the support of the transfer function
- Optical fibers → $A(\alpha, t) = c(t)G(\alpha)$
- Transform $N_s = (\frac{D}{r_0})^2$ random phases into 1 random flux
- Fibered (monomode) interferometers transmit the mean object spectrum over the pupil

→ the FOV is limited to one Airy disc

Data processing: the observables

- Object spatial spectrum → $O(f) = \frac{\int O(\alpha) e^{-2i\pi f \alpha} d\alpha}{\int O(\alpha) d\alpha}$

- The observables
  - The visibility → $V(f)$
  - The differential visibility → $V(f, \sigma_1)/V(f, \sigma_2)$
  - The differential phase → $\phi_d = \phi(\sigma_1) - \phi(\sigma_2)$
  - The closure phase → $\phi_c = \phi_{12} + \phi_{23} - \phi_{13}$

Data processing: starting point

- Data processing starting point, data corrected from
  - Detector bias
  - Flat field
  - Bad pixels
- The 3 basic signals per frame:
  - 1 interferometric signal → $S'(z) = S(z) + B(z)$
  - 2 photometric signals

$$P'_1(z) = K_1(z) + B_1(z) \qquad P'_2(z) = K_2(z) + B_2(z)$$

Interferometric equation

- Beam recombinaison → $$|\Psi_1(\sigma) + \Psi_2(\sigma)|^2 * g(\sigma)$$

- Interferometric equation: z = t (MIDI) or z = α (AMBER)

$$S'(z) = K_1(z) + K_2(z) + 2Vq(z+\Delta z, \delta\sigma)\sqrt{K_1(z)K_2(z)}cos\Phi(z) + B(z)$$

$$\Phi(z) = 2\pi f_c(z+\Delta z) + \phi_s + \phi_o$$

- Temporal coherence effect (assuming square spectral filter)

$$q(z+\Delta z, \delta\sigma) = sinc[f_c(z+\Delta z)\frac{\delta\sigma}{\sigma}]$$

Pre-processing: 1st order bias corrections

- Photometric bias corrections

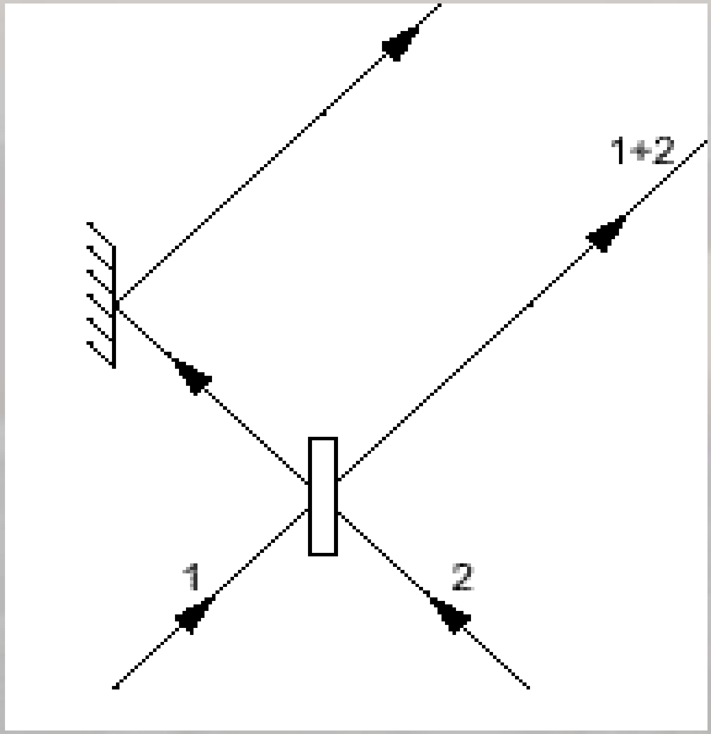
$$S(z) = S'(z) - < B(z) >$$

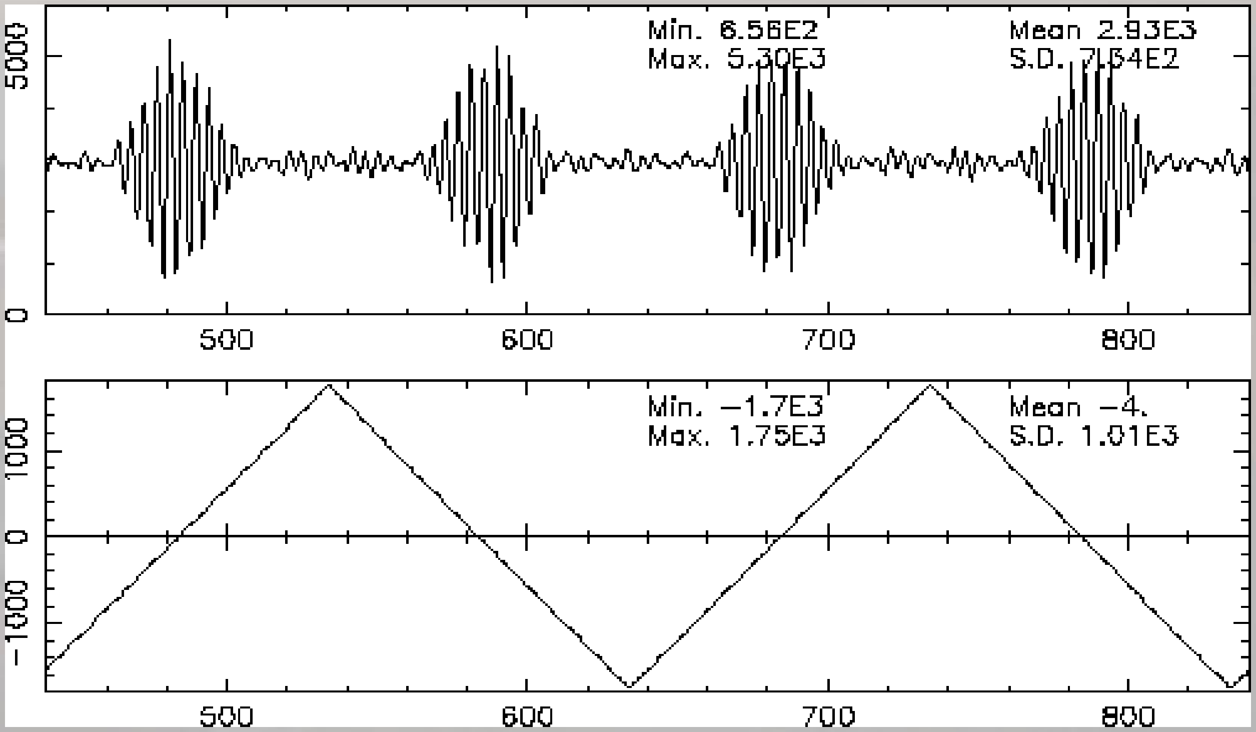
$$P_1(z) = P_1'(z) - < B_1(z) >$$

$$P_2(z) = P_2'(z) - < B_2(z) >$$

- The MIDI case

Temporal coding (MIDI)





Temporal coding: the visibility (1)

- Interferometric equation

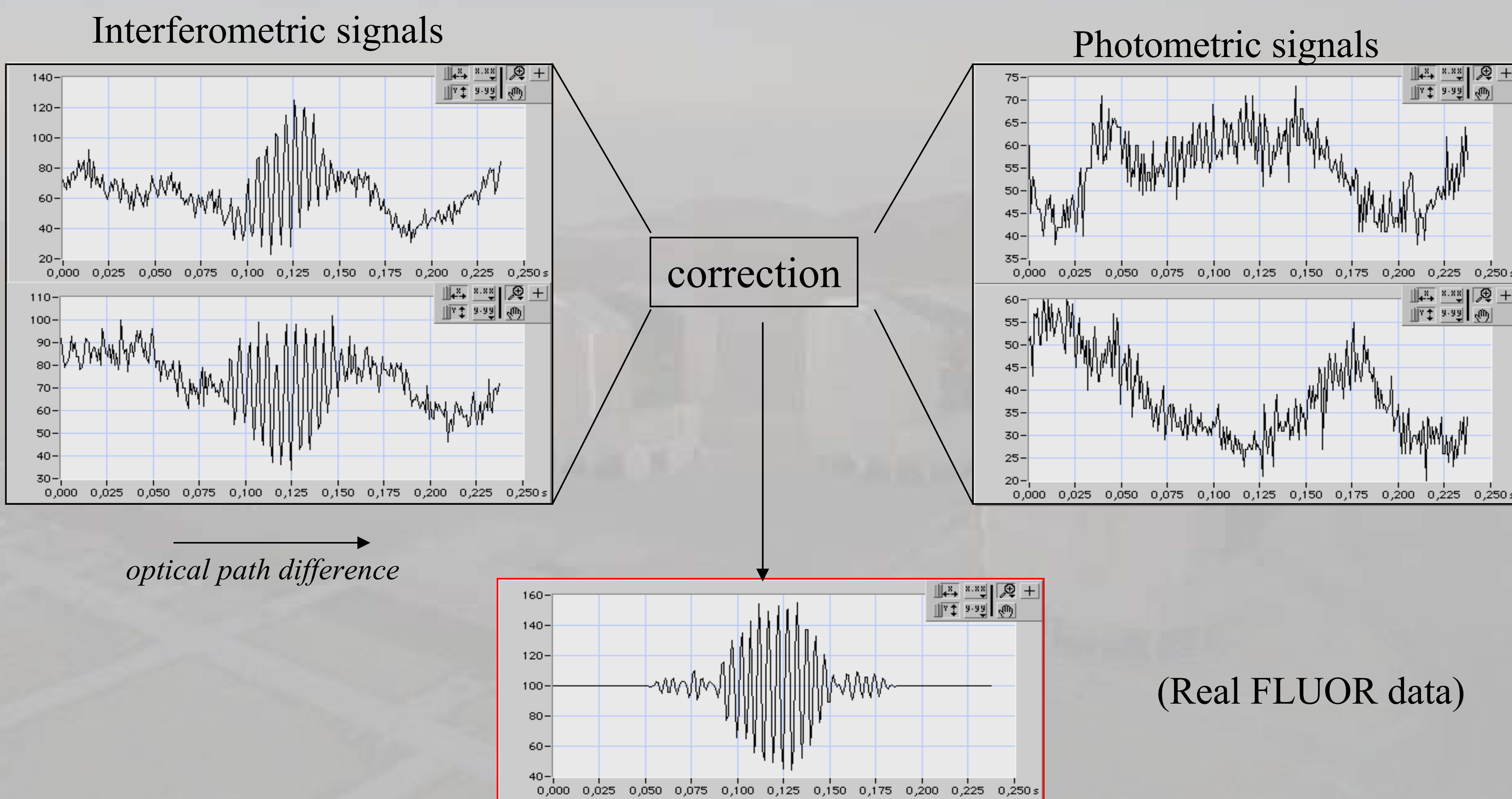
$$S(t) = K_1(t) + K_2(t) + 2V\sqrt{K_1(t)K_2(t)}\,sinc\left[(vt+d)\delta\sigma\right]cos\Phi(t)$$

$$\Phi(t) = 2\pi vt\sigma + 2\pi d\sigma + \phi_s + \phi_o$$

- Correction of the interferogram

$$s(t) = \frac{S(t) - P_1(t) - P_2(t)}{\sqrt{P_1(t)P_2(t)}} = 2V\,sinc\left[(vt+d)\delta\sigma\right]cos\Phi(t)$$

Temporal coding: an example of interferogram



(Real FLUOR data)

Temporal coding: the visibility extraction

- Spectrum → $s(\nu) = s^+(\nu) + s^-(\nu)$

$$< \int s^+(\nu)d\nu >= V e^{i\phi_0} e^{i\phi_s} < sinc(d\delta\sigma) e^{2i\pi d\sigma} >$$

Not a good estimator of the visibility

- Power spectrum → $< \int |s^+(\nu)|^2 d\nu >= V^2 + < b >$

Good estimator of the visibility → to be corrected for system visibility (calibrator)

- Avantage of temporal coding → insensitive to the OPD
- Inconvenient → sensitive to opd distorsions (relaxed at 10µm for MIDI)

Temporal coding: wavelet analysis

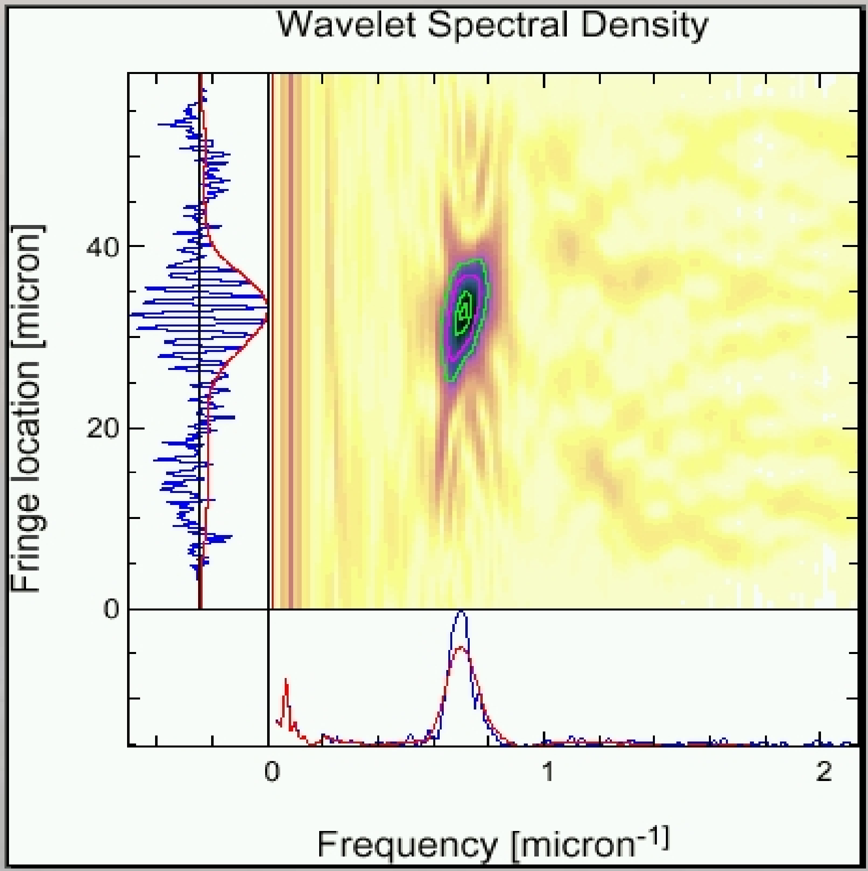
• Wavelet analysis could be a powerful tool to analyse temporal interferograms

• Allow to make a analysis of possible OPD variations along the interferogram

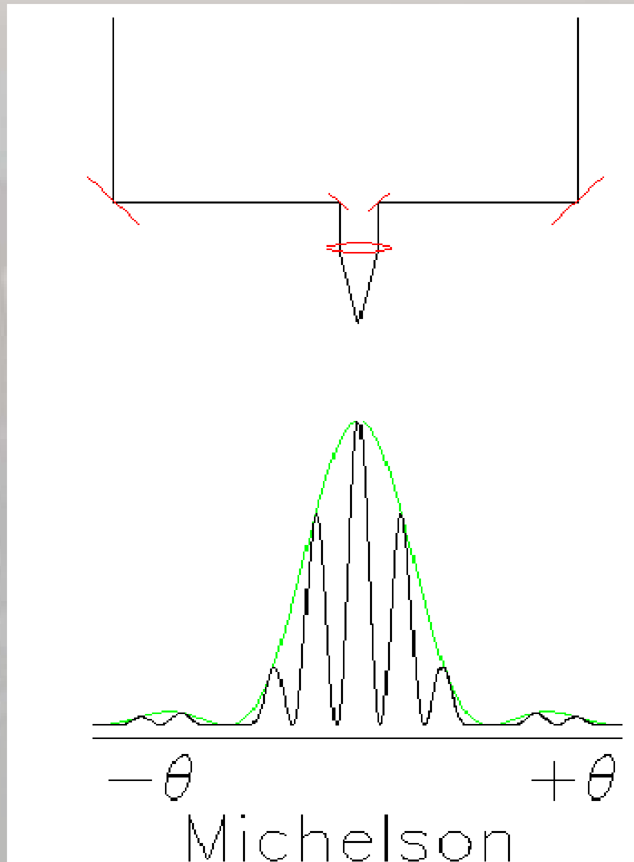
→ tool to test the quality

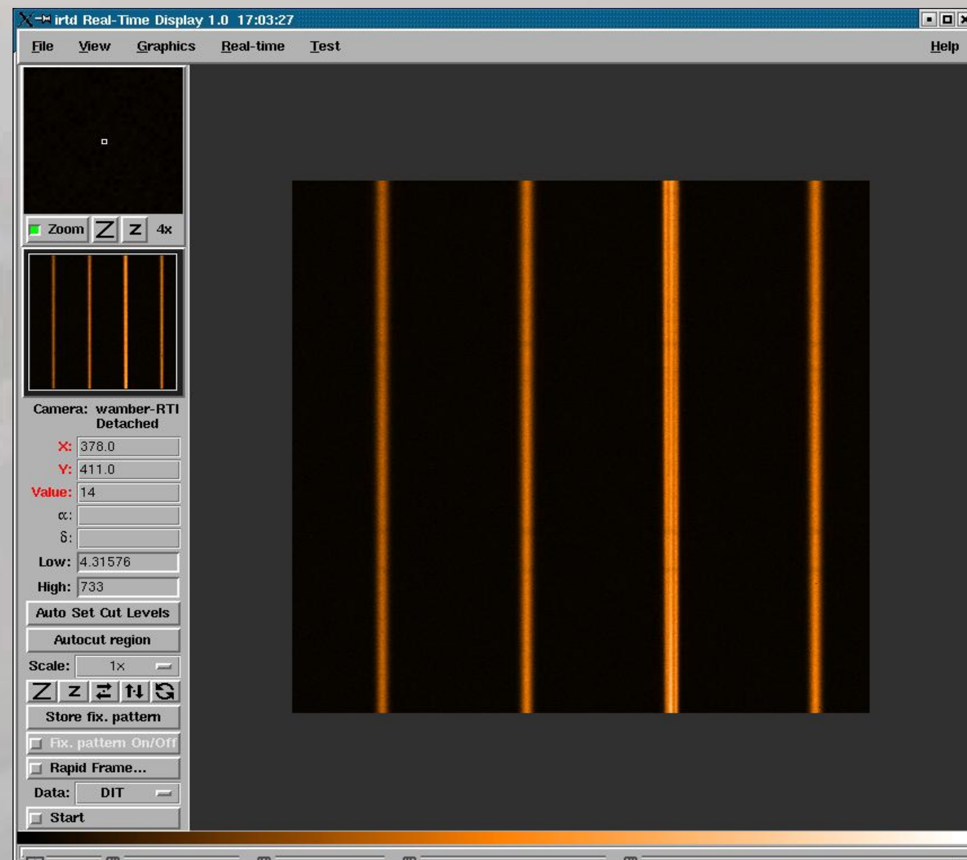
• Better estimate of the visibility?

→ to be investigated further



Spatial coding (standard)





Spatial coding: the visibility (1)

- Interferometric equation

$$S(\alpha) = K_1 a_1(\alpha) + K_2 a_2(\alpha) + 2V\sqrt{K_1 K_2}\sqrt{a_1(\alpha)a_2(\alpha)}\,sinc\big[(b\alpha + d)\delta\sigma\big]\,cos\Phi(\alpha)$$

$$\Phi(\alpha) = 2\pi b\alpha\sigma + 2\pi d\sigma + \phi_s + \phi_o$$

- Continuum correction

$$s(\alpha) = S(\alpha) - P_1(\alpha) - P_2(\alpha) = 2V\sqrt{K_1 K_2}\sqrt{a_1(\alpha)a_2(\alpha)}\,sinc\big[(b\alpha + d)\delta\sigma\big]\,cos\Phi(\alpha)$$

Spatial coding: the visibility (standard 1)

- Spectrum → $s(f) = s^{+}(f) + s^{-}(f)$

$$< \int s^{+}(f) df >= V e^{i\phi_o} e^{i\phi_s} \sqrt{a_1(0) a_2(0)} < \sqrt{K_1 K_2} sinc(d\delta\sigma) e^{2i\pi d\sigma} >$$

Not a good estimator of the visibility

- Power spectrum

$$< \int |s^{+}(f)|^2 df >= V^2 \int a_1(\alpha) a_2(\alpha) < K_1 K_2 sinc^2 \left[(b\alpha + d)\delta\sigma \right] > d\alpha$$

Sensitive to the OPD

Spatial coding: the visibility (standard 2)

- Necessity to correct each interferogram for the non zero OPD

- Phase of the interferogram $2\pi d\sigma + \phi_s + \phi_o$, linear function of the OPD

- d estimated from the slope of the fringes
  - differential phase of $s^{+}(f)$
  - includes also dispersion within fibers (assuming linearity with σ)

- Correction for the OPD

$$m(\alpha) = \frac{s(\alpha)}{sinc\left[(b\alpha + d)\delta\sigma\right]} = 2V\sqrt{K_1 K_2}\sqrt{a_1(\alpha)a_2(\alpha)}cos\Phi(\alpha)$$

Spatial coding: the visibility (standard 3)

- Power spectrum

$$< \int |m^+(f)|^2 df >= V^2 < K_1 K_2 > \int a_1(\alpha) a_2(\alpha) d\alpha + < b >$$

- Scalar product of the photometric channels

$$< \int P_1(\alpha) P_2(\alpha) d\alpha >=< K_1 K_2 > \int a_1(\alpha) a_2(\alpha) d\alpha$$

- The visibility $$V^2 = \frac{< \int |m^+(f)|^2 df > - < b >}{< \int P_1(\alpha) P_2(\alpha) d\alpha >}$$
  - correction of OPD jitter during integration easy

To be corrected for system visibility on calibrator

Spatial coding: the differential phase

- Cross spectrum at 2 different wavelengths

$$\int s^{+}(f,\sigma_1)df \int s^{+*}(f,\sigma_2)df = Ge^{2i\pi d(\sigma_1-\sigma_2)+i(\phi_1-\phi_2)+\phi_s}$$

- Non zero OPD estimated from the sope of the fringes as a function of σ

- Differential phase

$$\phi_1 - \phi_2 = Arg\left\langle e^{-2i\pi d(\sigma_1-\sigma_2)} \int s^{+}(f,\sigma_1)df \int s^{+*}(f,\sigma_2)df \right\rangle - \phi_s$$

- Correction of dispersion within fibers (2^{nd} order)

- System phase ϕ_s calibrated with the beam commuter device (bcd) and calibrator → problem of calibrators

Spatial coding: the closure phase (standard)

- With N telescopes the standard method for visibility and phase extraction is based on the separability of the Nx(N-1)/2 high frequency peaks

- The 3 telescopes case: 3 high frequency peaks in the spectrum

$$s^+(f - f_{12}) \quad s^+(f - f_{23}) \quad s^+(f - f_{13})$$

- Closure phase = bispectrum phase

$$\phi_{12} + \phi_{23} - \phi_{13} = Arg\left\langle \int s^+(f - f_{12})df \int s^+(f - f_{23})df \int s^{+*}(f - f_{13}df) \right\rangle - \phi_s$$

- The system phase ϕ_s (ideally zero) is to be calibrated on a reference source

The AMBER case

- The 2 telescopes case
  - the 2 high frequency peaks are separated
  - previous formalism applicable
- The 3 telescopes case
  - partial overlapping of the 3 frequency peaks to reduce the nb of pixels
  - previous formalism non applicable
  - need to work in the image plane by modelling each interferogram

The AMBER case: modelling the interferograms

- Continuum correction

$$m_k = S_k - P_{1k} - P_{2k} = 2V\sqrt{K_1 K_2}\sqrt{a_{1k}a_{2k}}\,sinc\big[(b\alpha_k + d)\delta\sigma\big]\,cos\Phi_k$$

$$\Phi_k = 2\pi b\alpha_k\sigma + 2\pi d\sigma + \phi_s + \phi_o$$

- Modelling the interferograms: need to know the OPD
  - Iterative process (3 or 4 steps)
  - 1st step: assume OPD = 0 and model the interferograms
  - 2nd step: 1st estimate the OPD from differential phase slope
  - 3rd step: inject estimated OPD and remodel the interferograms
  - 4th step: if necessary repeat 2nd and 3rd steps

How to model the interferograms

- Describing the 1st step → $m_k = c_k R - d_k I = \frac{1}{2}(W_k C + W_k^* C^*)$

- $C = R + iI$ is the complex coherent energy of the object

$$C = 2V\sqrt{K_1 K_2}\sqrt{\sum_{j=1}^{n} a_{1j} a_{2j}} \times e^{i\phi_o} e^{2i\pi d\sigma}$$

- $W_k = c_k + i d_k$ is the carrying wave of the interferometer

$$W_k = \sqrt{\frac{a_{1k} a_{2k}}{\sum_{j=1}^{n} a_{1j} a_{2j}}} \times e^{i(2\pi f \alpha_k + \phi_s)}$$

- The carrying wave is determined experimentally before the observations

The complex coherent energy

- C is estimated by minimizing → $\chi^2 = \sum_{k=1}^{n} \left(\frac{m_k - c_k R + d_k I}{\sigma_k} \right)^2$

- Which gives:

$$R = \frac{\sum_{j=1}^{n} \frac{c_k m_k}{\sigma_k^2} \sum_{j=1}^{n} \frac{d_k^2}{\sigma_k^2} - \sum_{j=1}^{n} \frac{d_k m_k}{\sigma_k^2} \sum_{j=1}^{n} \frac{c_k d_k}{\sigma_k^2}}{\sum_{j=1}^{n} \frac{c_k^2}{\sigma_k^2} \sum_{j=1}^{n} \frac{d_k^2}{\sigma_k^2} - \left(\sum_{j=1}^{n} \frac{c_k d_k}{\sigma_k^2} \right)^2}$$

$$I = \frac{\sum_{j=1}^{n} \frac{c_k m_k}{\sigma_k^2} \sum_{j=1}^{n} \frac{c_k d_k}{\sigma_k^2} - \sum_{j=1}^{n} \frac{d_k m_k}{\sigma_k^2} \sum_{j=1}^{n} \frac{c_k^2}{\sigma_k^2}}{\sum_{j=1}^{n} \frac{c_k^2}{\sigma_k^2} \sum_{j=1}^{n} \frac{d_k^2}{\sigma_k^2} - \left(\sum_{j=1}^{n} \frac{c_k d_k}{\sigma_k^2} \right)^2}$$

- C is obtained by matrix transformation → $\begin{bmatrix} R \\ I \end{bmatrix} =$ [m] x [P2VM]

- Generalisation to n telescopes immediate

Estimates of the observables

- The visibility → $$V^2 = \frac{<|C|^2> - <b>}{4 < \sum_{j=1}^{n} P_{1k}P_{2k} >} = \frac{<R^2> + <I^2> - <b>}{4 < \sum_{j=1}^{n} P_{1k}P_{2k} >}$$

- The differential visibility → $$C_1 C_2^* = G e^{2i\pi d(\sigma_1 - \sigma_2) + i(\phi_1 - \phi_2) + \phi_s}$$

- The differential phase → $$\phi_1 - \phi_2 = Arg\left\langle e^{-2i\pi d(\sigma_1 - \sigma_2)} C_1 C_2^* \right\rangle - \phi_s$$

- The closure phase → $$\phi_{12} + \phi_{23} - \phi_{13} = Arg\left\langle C_{12} C_{23} C_{13}^* \right\rangle - \phi_s$$