Introduction to interferometry

EuroWinter School

Observing with the Very Large Telescope Interferometer

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Preamble

- Learning interferometry is like learning to ski:
  - You have to want to.
  - You start on the green slopes.
  - Having expensive skis or reading about it doesn’t help.
  - You don’t have to know how to make skis (but it can help).
  - It may not help you to escape a shark.

- This is a school:
  - Assume nothing, as I will!
  - We have a lot to cover – this will not be easy.
  - Knowing what questions to ask is what is important.
  - Use the lecturers.
  - Trust me.

- I am not a car salesman.

Outline

- Image formation with conventional telescopes
  - Incoherent imaging equation
  - Fourier decomposition
- Coherence functions
  - Temporal coherence
  - Spatial coherence
- Interferometric measurements
  - Fringe parameters
- Imaging with interferometers
  - Rules of thumb
  - Interferometric images
  - Sensitivity

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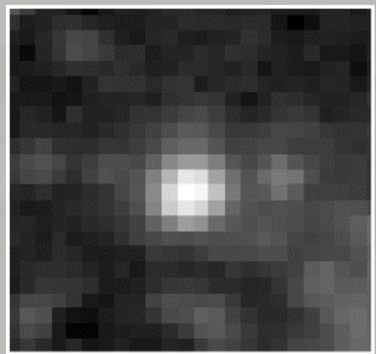
Image formation with conventional telescopes

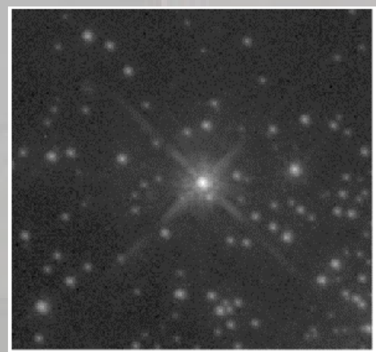
Fundamental relationship for incoherent space-invariant imaging:

$$I(l, m) = \iint P(l-l', m-m')\, O(l', m')\, dl'\, dm' ,$$

i.e. the observed brightness distribution is the true source brightness distribution convolved with a point-spread function, P(l, m).

Note that here l and m are angular coordinates on the sky, measured in radians.





An alternative representation

This convolutional relationship, which typifies the behaviour of linear space-invariant (isoplanatic) systems, can be written alternatively, by taking the Fourier transform of each side of the equation, as:

$$I(u, v) = T(u, v) \times O(u, v) ,$$

where *italic* functions refer to the Fourier transforms of their roman counterparts, and u and v are now spatial frequencies measured in radians^{-1}.

Importantly, the essential properties of the imaging system are encapsulated in a complex multiplicative transfer function, $T(u, v)$.

The Transfer function

In general the transfer function is obtained from the auto-correlation of the complex pupil function:

$$T(u, v) = \iint P^*(x, y)\, P(x+u, y+v)\, dx\, dy \,,$$

where x and y denote co-ordinates in the pupil.

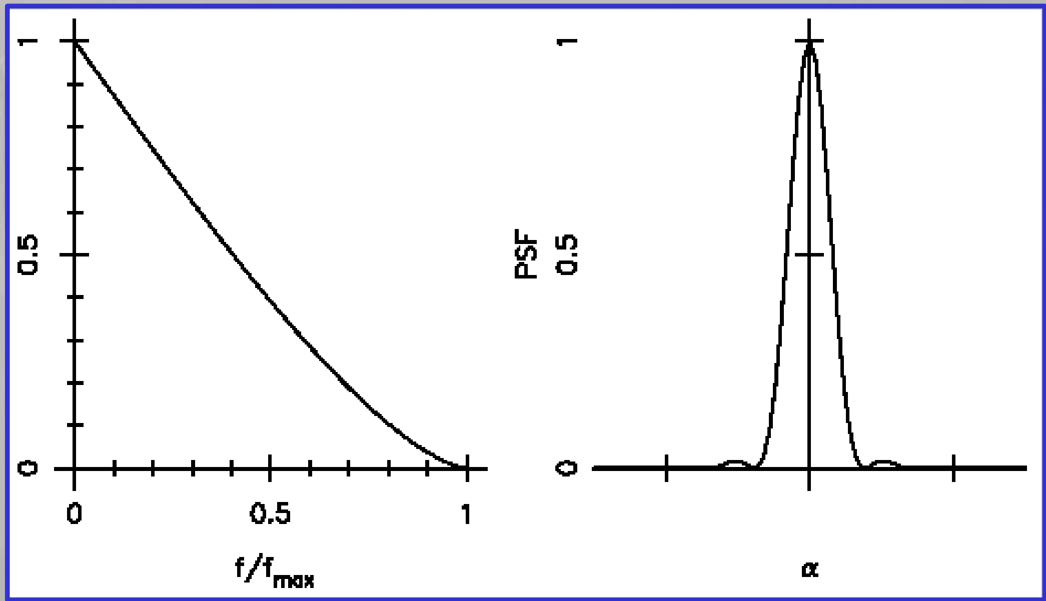
A number of key features of this formalism are worth noting:

- For each spatial frequency, *u*, there is a physical baseline, B, in the pupil, of length λu.
- In the absence of aberrations P(x, y) is equal to 1 where the aperture is transmitting and 0 otherwise.
- For a circularly symmetric aperture, the transfer function can be written as a function of a single co-ordinate: $T(f)$, with $f^2 = u^2 + v^2$.

An example

As an example its is useful to consider the normalized transfer function and point-spread function of a circular pupil with no central obscuration:

- $T(f)$ falls smoothly to zero at $f_{max} = D/\lambda$.
- The PSF is the familiar Airy pattern.
- The full-width at half-maximum of this is at approximately $0.9\ \lambda/D$.



What should we draw from all this?

- Decomposition of an image into a series of spatially separated compact PSFs.
- The equivalence of this to a superposition of non-localized co-sinusoids.
- The description of an image in terms of its Fourier components.
- The action of an incoherent imaging system as a filter for the true spatial Fourier spectrum of the source.
- The association of each Fourier component (or spatial frequency) with a distinct physical baseline in the aperture that samples the light.
- The form of the point-spread function as arising from the relative sampling (and hence weighting given to) the different spatial frequencies (and hence baselines) measured by the pupil of the imaging system.

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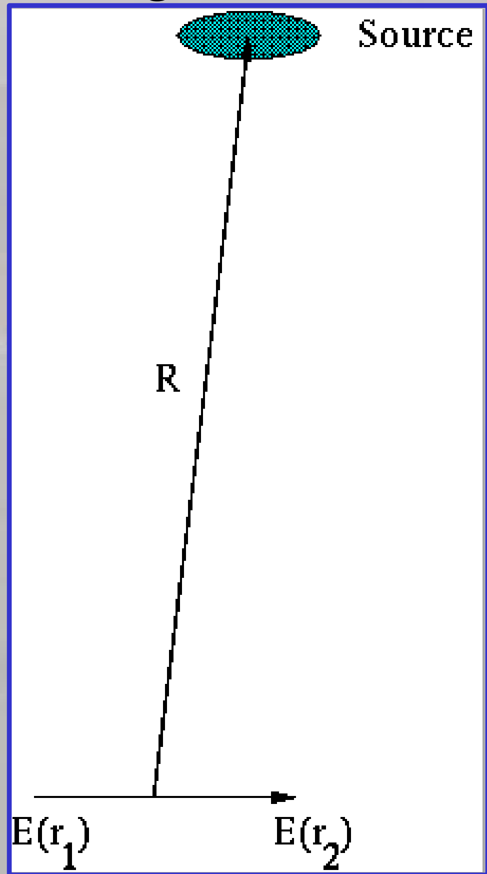
Coherence functions

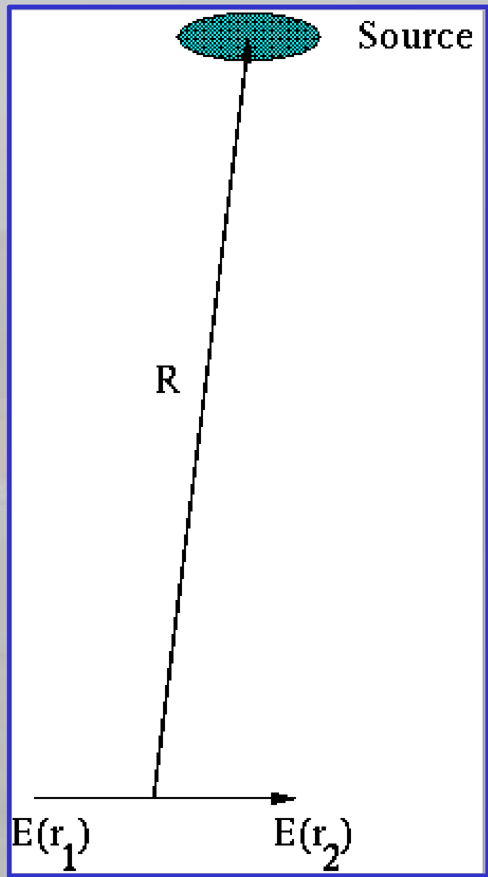
In the context of interferometric imaging it is sometimes useful to consider the spatio-temporal correlations of the field arising from an astronomical source:



- This means measuring the electric field produced by the source at some locations and looking at the correlations between these measured fields.
- The reason for doing this is that the spectral and spatial properties of the source can, in principle, be recovered from these measurements without using any other apparatus.
- This is imaging without a telescope!

The temporal and spatial coherence functions

- Measure electric field from a distant source at two locations r_1 and r_2 at times t_1 and t_2.
- Each field is composed of contributions from each element of the source.
- We define the spatio-temporal coherence function as $V(r_1, t_1, r_2, t_2) = \langle E(r_1, t_1) \times E^*(r_2, t_2) \rangle$.
- We are interested in two special cases:
  - $t_1 = t_2$: spatial coherence function.
  - $r_1 = r_2$: temporal coherence function.



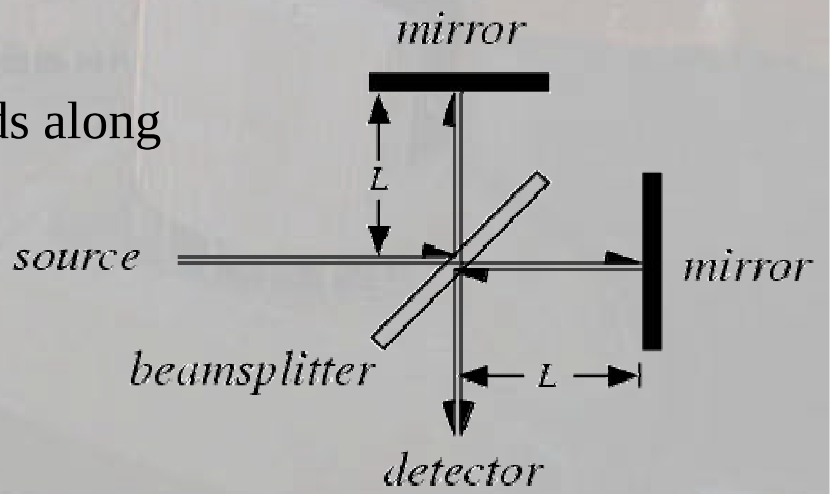
The temporal coherence function

For astronomical sources, this coherence function can be written as:

$$\langle E(r_1, t_1) \times E^{*}(r_1, t_2) \rangle = V(t_1 - t_2) = V(\tau) \ .$$

In this case we should note that:

- The coherence function does not depend on r_1.
- It is a function of a time delay, $\tau = t_1 - t_2$.
- It quantifies the extent to which the fields along a give wave train are correlated.
- It is related to the quantity that a laboratory Michelson interferometer measures.



Use of the temporal coherence function

The importance of the temporal coherence function arises from an important result in physics, the Wiener-Khintchine theorem.

This says that the normalized value of the temporal coherence function $V(\tau)$ is equal to the normalized Fourier transform of the spectral energy distribution, $B(\omega)$, of the source:

$$V(\tau) = \int B(\omega)\, e^{-i\omega\tau}\, d\omega \,/ \int B(\omega) d\omega \, .$$

- A broad spectral energy distribution leads to a coherence function that decays rapidly since τ and ω are reciprocal coordinates.
- We can define a coherence time: $\tau_{coh} \sim 1/\Delta\nu$, with $\Delta\nu = \Delta\omega/2\pi$ the spectral bandwidth of the radiation.

Measurements of $V(\tau)$ allow recovery of the source spectrum.

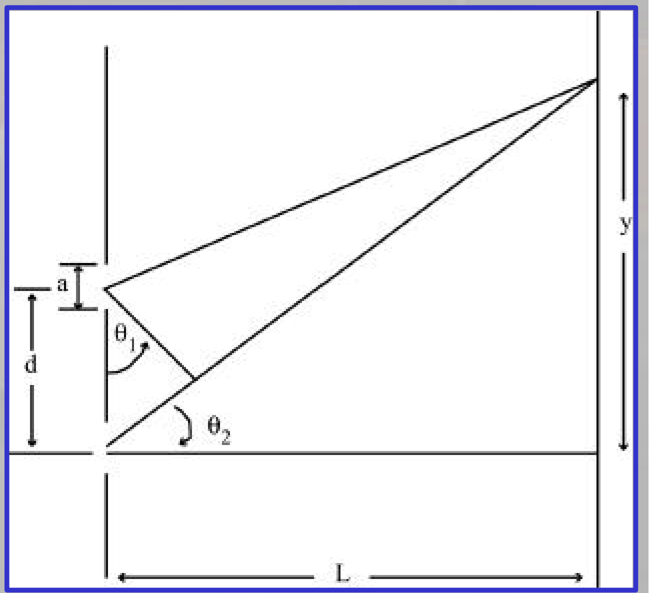
The spatial coherence function

For astronomical sources, this coherence function can be written as:

$$\langle E(r_1, t_1) \times E^*(r_2, t_1) \rangle = V(r_1 - r_2) = V(\rho) \ .$$

In this case we see that:

- This coherence function does not depend on t_1.
- It is a function of a vector separation, $\underline{\rho} = r_1 - r_2$.
- It quantifies the correlations between different spatial locations on a wavefront.
- It corresponds to the quantity that a Young's slit experiment investigates (on axis).



Use of the spatial coherence function

The importance of the spatial coherence function arises from another important result in physics, the van Cittert-Zernike theorem.

This states that, for incoherent sources in the far-field, the normalized value of the spatial coherence function $V(\underline{\rho})$ is equal to the normalized Fourier transform of the brightness distribution in the sky, $I(\underline{\alpha})$:

$$V(\underline{\rho}) = \int I(\underline{\alpha})\, e^{-i\, 2\pi/\lambda\, (\underline{\alpha}.\underline{\rho})}\, d\underline{\rho} \;/\; \int I(\underline{\alpha})\, d\underline{\rho}\ ,$$

or in slightly different notation:

$$V(u, v) = \iint I(l, m)\, e^{-i2\pi(ul + vm)}\, dl\, dm \;/\; \iint I(l, m)\, dl\, dm\ ,$$

where u and v are the components of the baseline $\underline{\rho}$ measured in wavelengths, and l and m are angular coordinates on the sky.

What should we draw from all this?

- Measurements of these coherence functions allow us to interrogate a source without using a conventional imaging telescope.

- This in turn relies upon access to measurements of time-averaged products of field quantities like $\langle E(r_1) \times E^*(r_2) \rangle$.

- The relationships between the source parameters and the coherence functions is a Fourier transform. Hence it is:
  - Linear.
  - Invertible.
  - Complex.

- We note the mathematical equivalence of the spatial coherence function $V(\tau=0, \rho)$ and the Fourier decomposition of an image we referred to earlier.

Spatial interferometry

We can put this all together in the following form:

- We can describe a source in the sky as a superposition of co-sinusoids, each of which corresponds to a given spatial frequency.
- Measurements of the coherence function are in fact measurements of the strength of each of these Fourier components.
- Interferometers are merely devices to measure the coherence function.
- Two telescopes with a projected separation B will measure the value of the Fourier transform of the source brightness distribution at a spatial frequency $u = B/\lambda$.
- Telescopes do all of this for you for a range of baselines at once for free!

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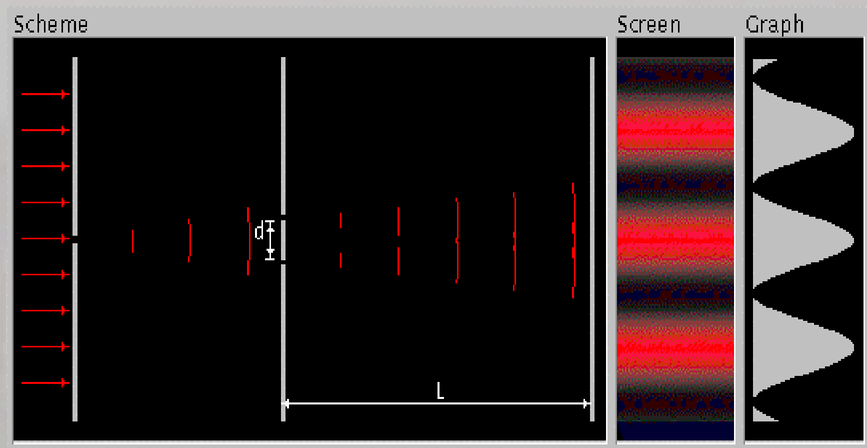
An aside on measurement

In what sense do laboratory set-ups like a Michelson or Young's slit experiment measure coherence functions?



- The detector receives contributions from each slit, E_1 and E_2.
- The fields are added: E_1+E_2.
- The time averaged intensity is measured:

á$(E_1+E_2)\times(E_1+E_2)^*$ñ = á$|E_1|^2$ñ + á$|E_2|^2$ñ + á$E_1E_2^*$ñ + á$E_2E_1^*$ñ

= á $|E_1|^2$ñ + á $|E_2|^2$ñ + á $2|E_1||E_2|\cos(\varphi)$ ñ

where φ is the phase difference between E_1 and E_2.

So the properties of the fringe pattern encode the coherence function.

Measurements of fringes

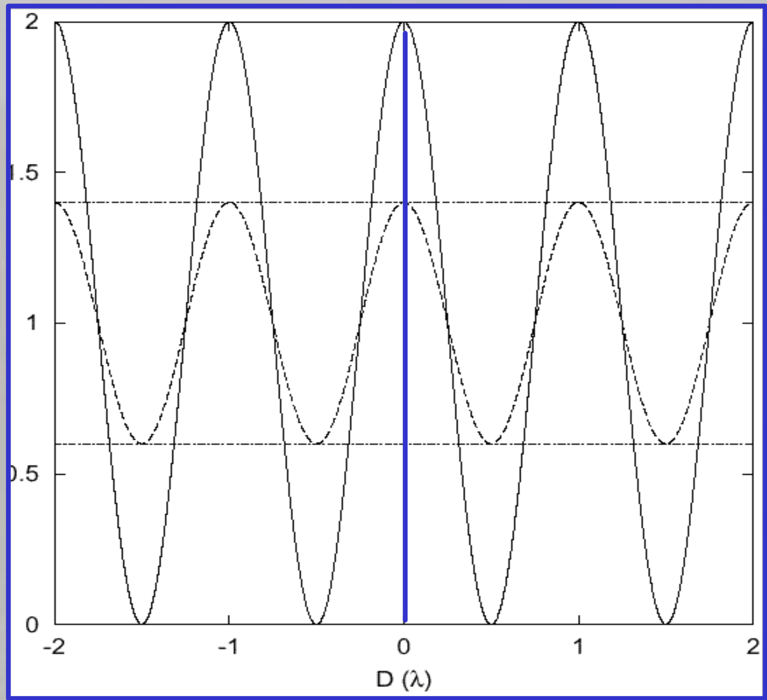
From an interferometric point of view the key features of any interference fringe are its modulation and its location with respect to some reference point.

In particular we can identify:

- The fringe visibility:

$$V = \frac{[I_{max} - I_{min}]}{[I_{max} + I_{min}]}$$

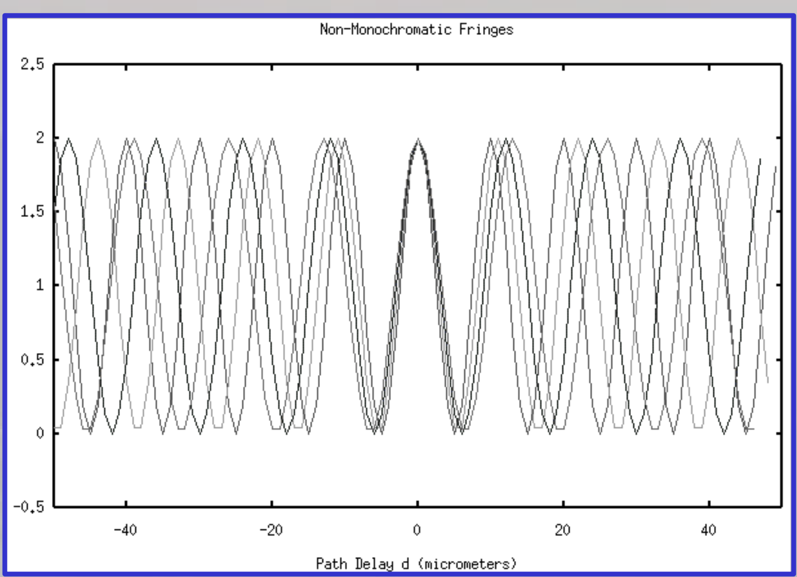
- The fringe phase:
  - The location of the white-light fringe as measured from some reference (radians).

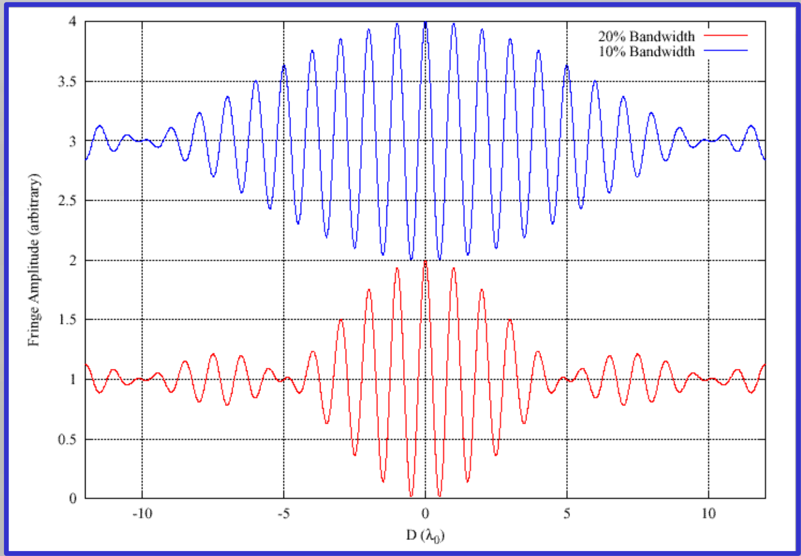


These measure the amplitude and phase of the complex coherence function, respectively.

Temporal coherence revisited

Consider the response of a Michelson interferometer to a range of wavelengths:





Gives a resulting fringe pattern whose modulation depth decreases as the delay between the interfering beams increase.

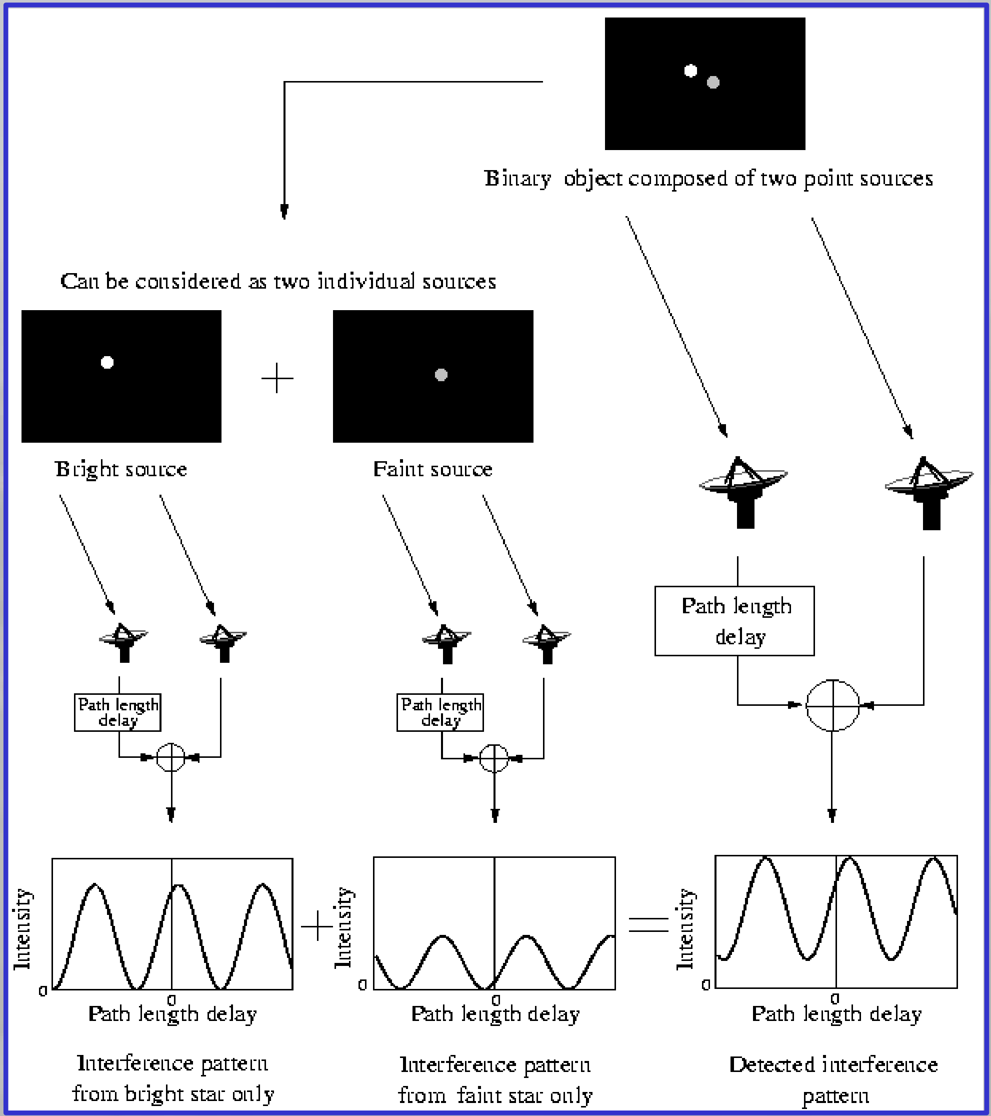
Note the fringe modulation disappears when the delay, $D = \lambda^2/\Delta\lambda = l_{coh}$.

Spatial coherence revisited

Consider the response of an two-element interferometer observing a star comprising two separate infinitesimally small sources.

As before, the resulting fringe pattern has a modulation depth that is reduced with respect to that from each source individually.

Note how the positions of the sources are encoded in the fringe phase.



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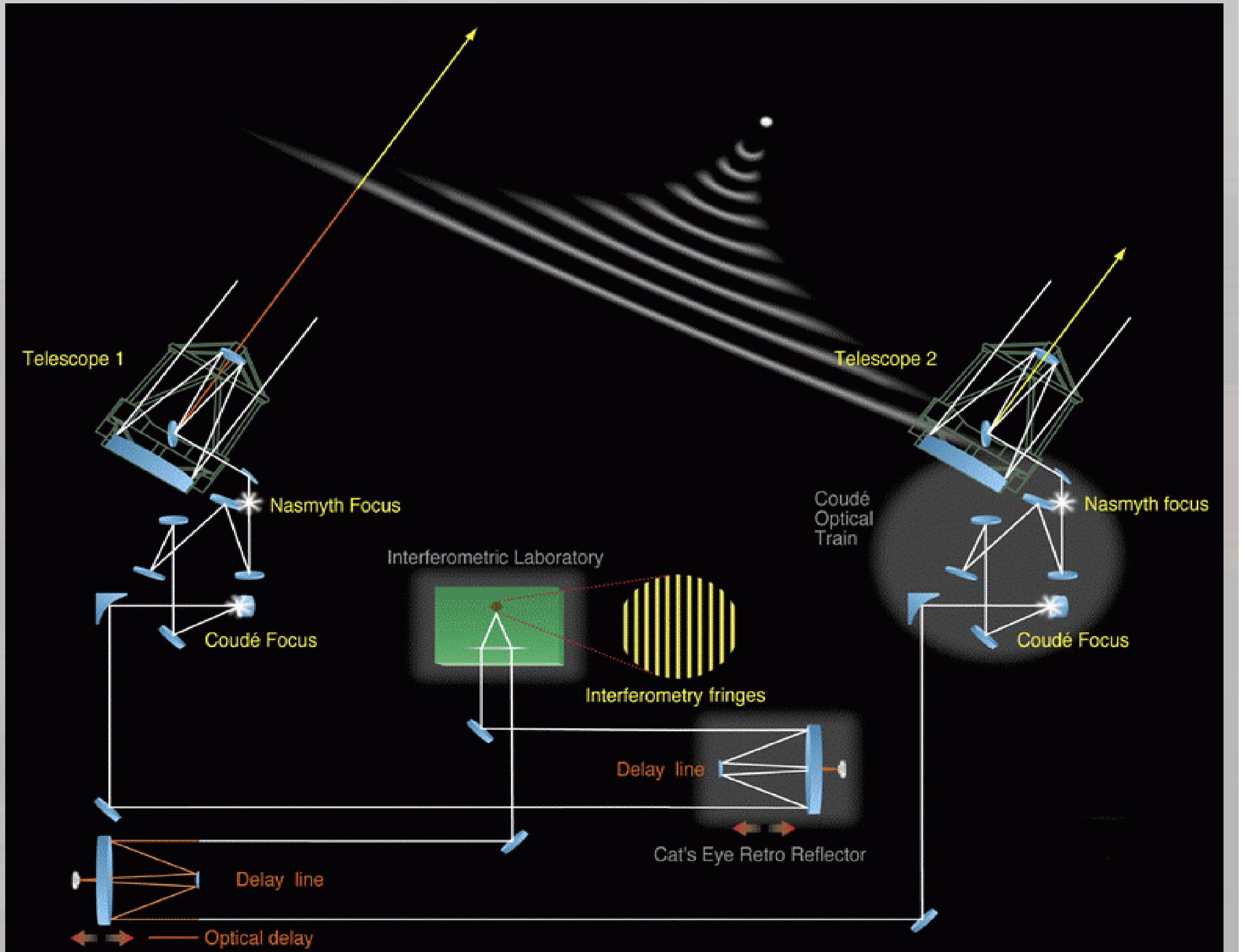
A reality check

How is all this related to the VLTI?

- Telescopes sample the fields at r_1 and r_2.
- Optical train delivers the radiation to a laboratory.
- Delay lines assure that we measure when $t_1=t_2$.
- The instruments mix the beams and detect the fringes.



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Imaging with interferometers

Physical basis is the van Cittert-Zernike theorem:

- Fourier transform of the brightness distribution is the coherence, or visibility function, $V(u, v) = V(B_x/\lambda,\ B_y/\lambda)$

So in principle the strategy is straightforward:

- Measure V for as many values of B as possible.
- Perform an inverse Fourier transform $\Rightarrow$image of the source.

But we need to consider the following topics:

- Typical visibility functions - what do they look like?
- How complete do the measurement of *V(u, v)* have to be?
- What is the nature of the images that can be recovered?

[Note that all of this will assume the absence of a turbulent atmosphere.]

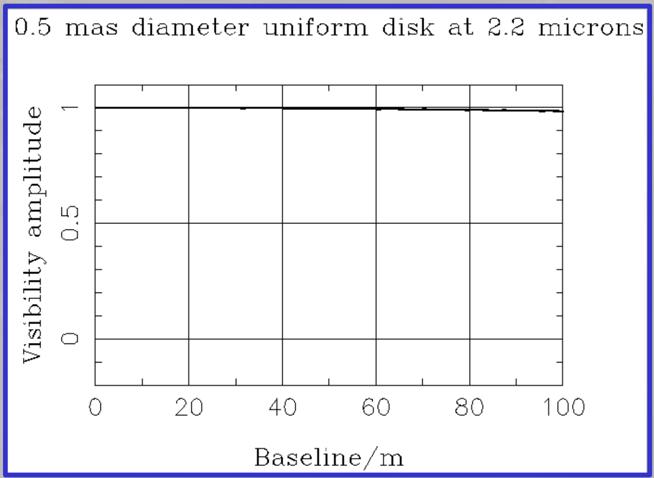
Simple sources (i)

$$V(u) = \int I(l)\, e^{-i2\pi(ul)}\, dl \,/\, \int I(l)\, dl$$

[Note that here we explore one-dimensional examples for simplicity.]

- Point source of strength A_1 and located at angle l_1 relative to the optical axis.

$$V(u) = \int A_1\delta(l-l_1)\, e^{-i2\pi(ul)}\, dl \,/\, \int A_1\delta(l-l_1)dl$$
$$= e^{-i2\pi(ul_1)}$$



- The visibility amplitude is unity $\forall\, u$.
- The visibility phase varies linearly with u (= B/λ).
- Sources such as this are easy to observe, but of little interest if you've built an interferometer for high-angular resolution imaging.

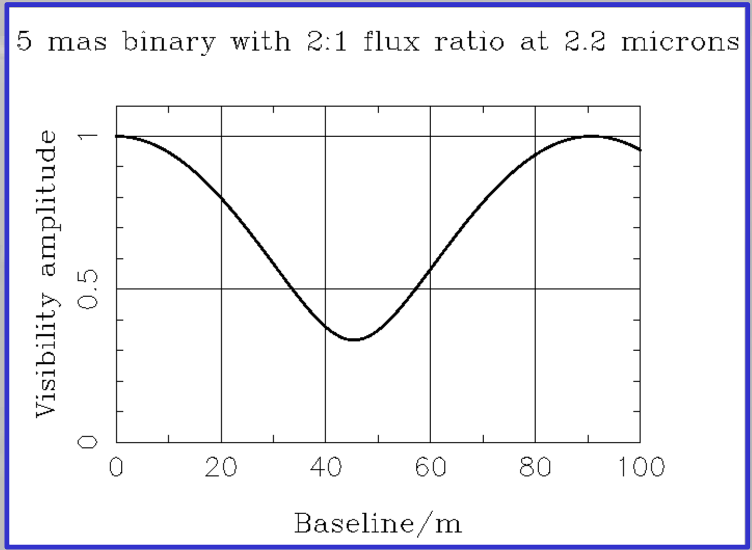
Simple sources (ii)

A double source comprising point sources of strength A_1 and A_2 located at angles 0 and l_2 relative to the optical axis.

$$V(u) = \int [A_1\delta(l) + A_2\delta(l-l_2)]\, e^{-i2\pi(ul)}\, dl \,/\, \int [A_1\delta(l) + A_2\delta(l-l_2)]\, dl$$

$$= [A_1+A_2 e^{-i2\pi(ul_2)}] / [A_1+A_2]$$

- The visibility amplitude and phase oscillate as functions of *u*.
- To identify this as a binary, baselines from $0 \rightarrow \lambda/l_2$ are required.



If the ratio of component fluxes is large the modulation of the visibility becomes increasingly difficult to measure.

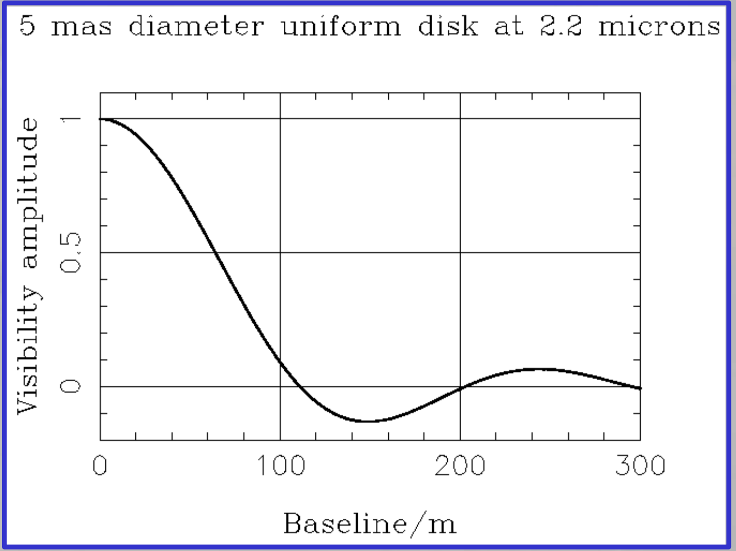
Simple sources (iii)

A uniform on-axis disc source of diameter θ.

$$V(u_r) \propto \int^{\theta/2} \rho\, J_0(2\pi\rho u_r)\, d\rho$$

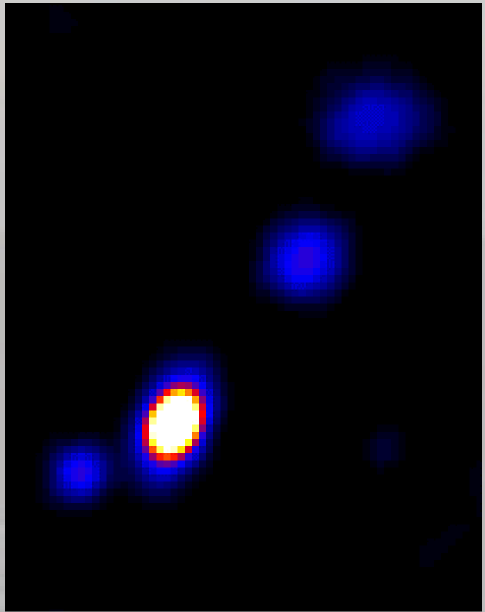
$$= 2J_1(\pi\theta u_r) / (\pi\theta u_r)$$

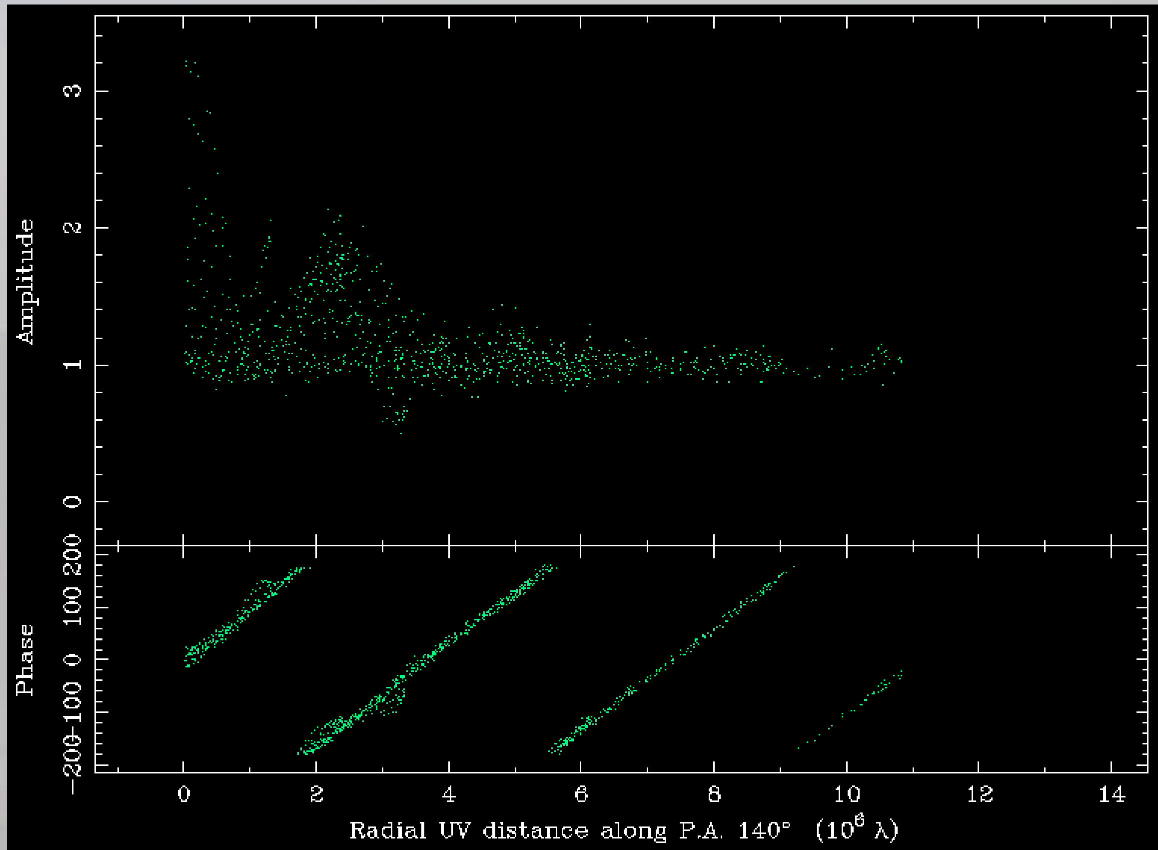
- To identify this as a disc requires baselines from $0 \rightarrow \lambda/\theta$ at least.
- The visibility amplitude falls rapidly as u_r increases.



Information on scales smaller than the disc diameter correspond to values of u_r where V << 1, and is hence difficult to measure.

A more complicated example





Sources without a significant component that is unresolved by the interferometer will have visibility functions that fall close to zero rapidly, and hence be difficult to image with many resolution elements across their total extent.

Image reconstruction

We start with the fundamental relationship between the visibility function and the normalized sky brightness:

$$I_{norm}(l, m) = \iint V(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv$$

In practice what we measure is a sampled version of *V(u, v)*, so the image we have access to is to the so-called "dirty map":

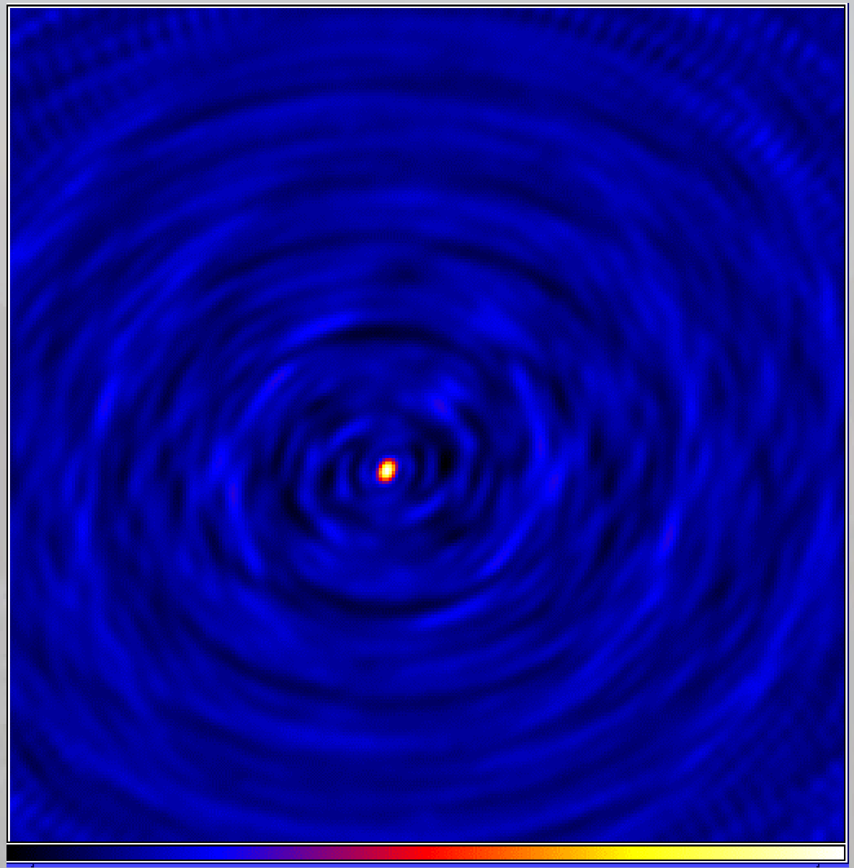
$$I_{dirty}(l, m) = \iint S(u, v)\, V(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv$$

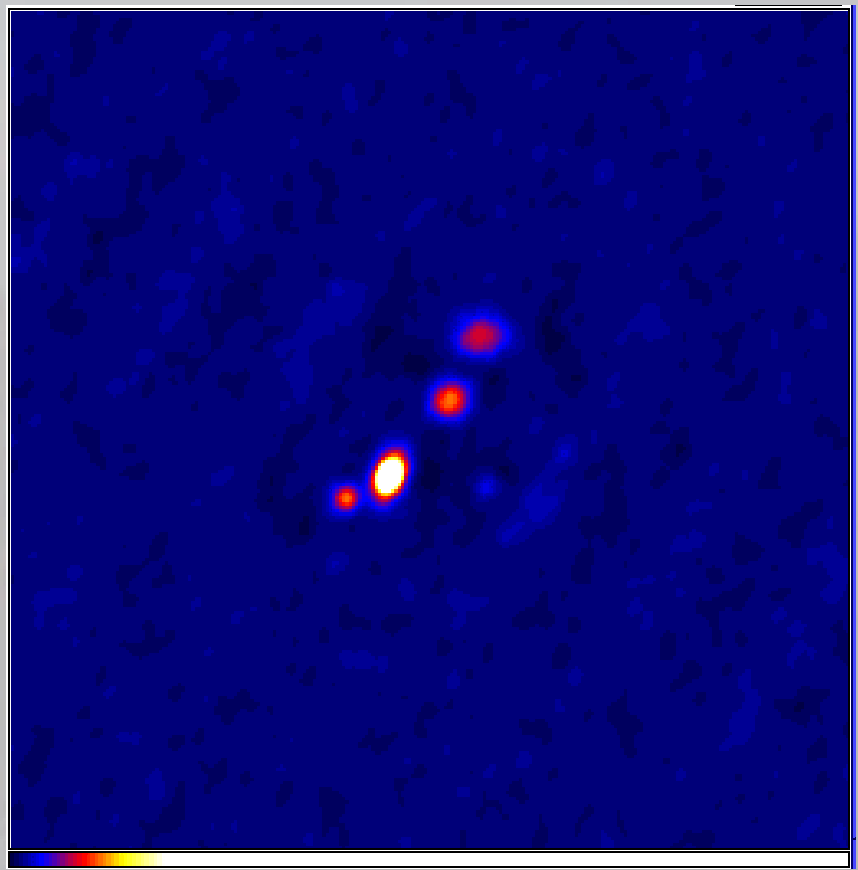
$$= B_{dirty}(l, m) * I_{norm}(l, m)\ ,$$

where $B_{dirty}(l,m)$ is the Fourier transform of the sampling distribution, or dirty-beam.

The dirty-beam is the interferometer PSF, and while it in general is far less attractive than an Airy pattern, it's shape is completely determined by the samples of the visibility function that are measured.

Deconvolution in interferometry





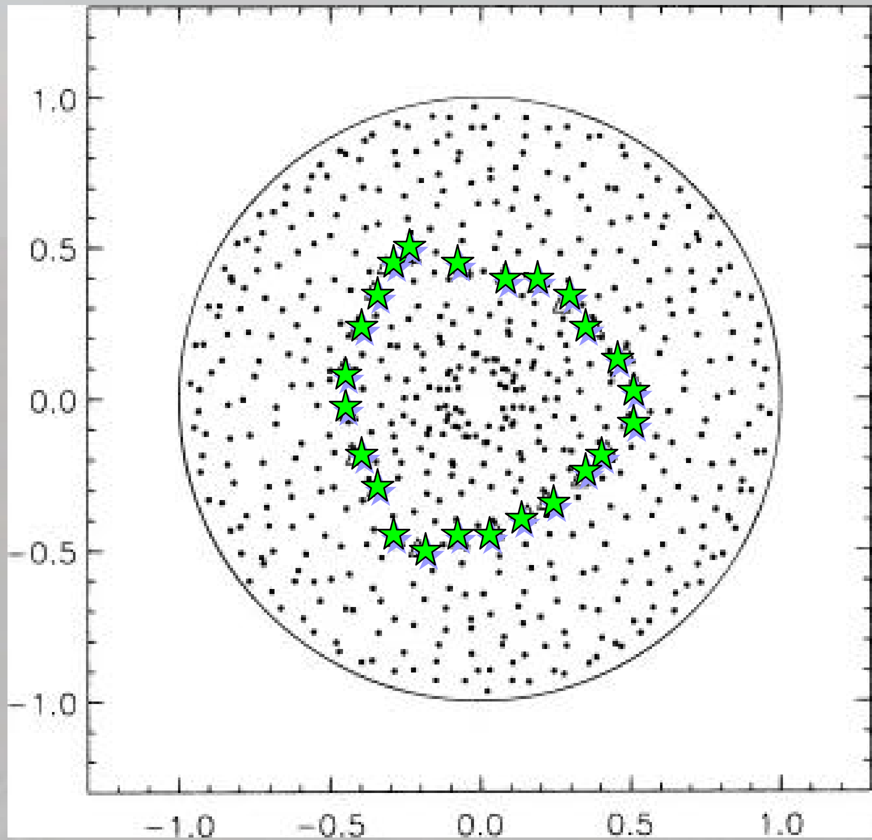
Correcting an interferometric map for the Fourier plane sampling function is known as deconvolution (CLEAN, MEM, WIPE).

Important rules of thumb

- The number of visibility data ≥ number of filled pixels in the recovered image:
  - $N(N-1)/2 \times$ number of reconfigurations ≥ number of filled pixels.
- The distribution of samples should be as uniform as possible:
  - To aid the deconvolution process.
- The range of interferometer baselines, i.e. B_{max}/B_{min}, will govern the range of spatial scales in the map.
- There is no need to sample the visibility function too finely:
  - For a source of maximum extent θ_{max}, sampling very much finer than $\Delta u \sim 1/\theta_{max}$ is unnecessary.

UV coverage

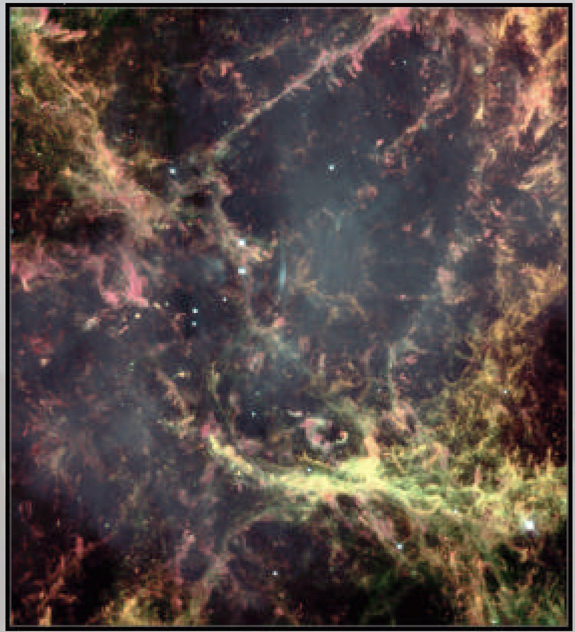
- An example of the instantaneous Fourier plane, or (*u*, *v*), coverage of a 24-element "Keto" array.
- The telescope locations are denoted by the stars, and the baselines (276 in total) by the dots.
- Broadly speaking this gives "uniform" sampling, save for a clustering of baselines near to the origin.
- The main shortcoming is the central hole near the origin.

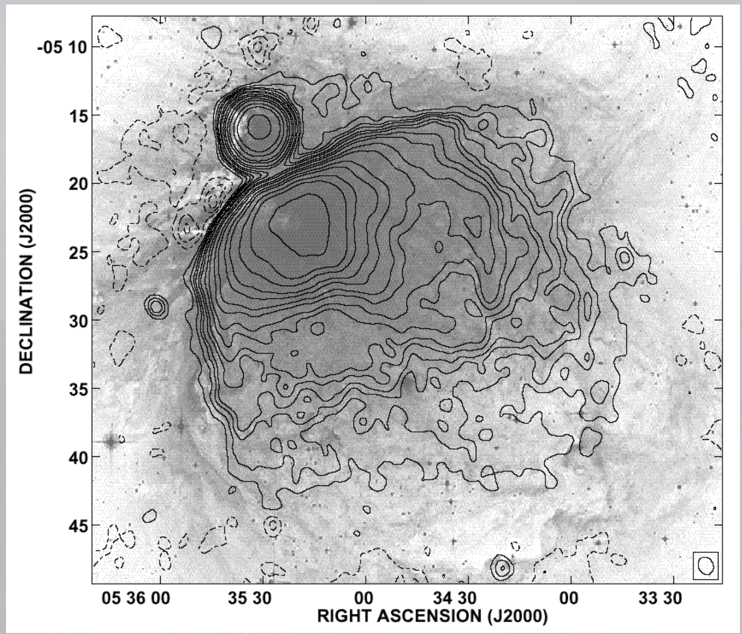


Field of view and image quality

- The FOV will depend upon:
  - The field of view of the individual collectors. This is often referred to as the primary beam.
  - The FOV seen by the detectors. This is limited by vignetting along the optical train.
  - The spectral resolution. The interference condition OPD < $\lambda^2/\Delta\lambda$ must be satisfied for all field angles. Generally ⇒ $FOV \leq [\lambda/B][\lambda/\Delta\lambda]$.
- Dynamic range:
  - The ratio of maximum intensity to the weakest believable intensity in the image.
  - > 10^5:1 is achievable in the very best radio images, but of order several × 100:1 is more usual.
  - $DR \sim [S/N]_{per\text{-}datum} \times [N_{data}]^{1/2}$
- Fidelity:
  - Difficult to quantify, but clearly dependent on the completeness of the Fourier plane sampling.

Conventional vs. interferometric imaging





Optical HST (left) and 330Mhz VLA (right) images of the Crab Nebula and the Orion nebula. Note the differences in the:

- Range of spatial scales in each image.
- The range of intensities.
- The complexity of each image.
- The field of view as measured in resolution elements.

Sensitivity

What does this actually mean in an optical/infrared interferometric context?

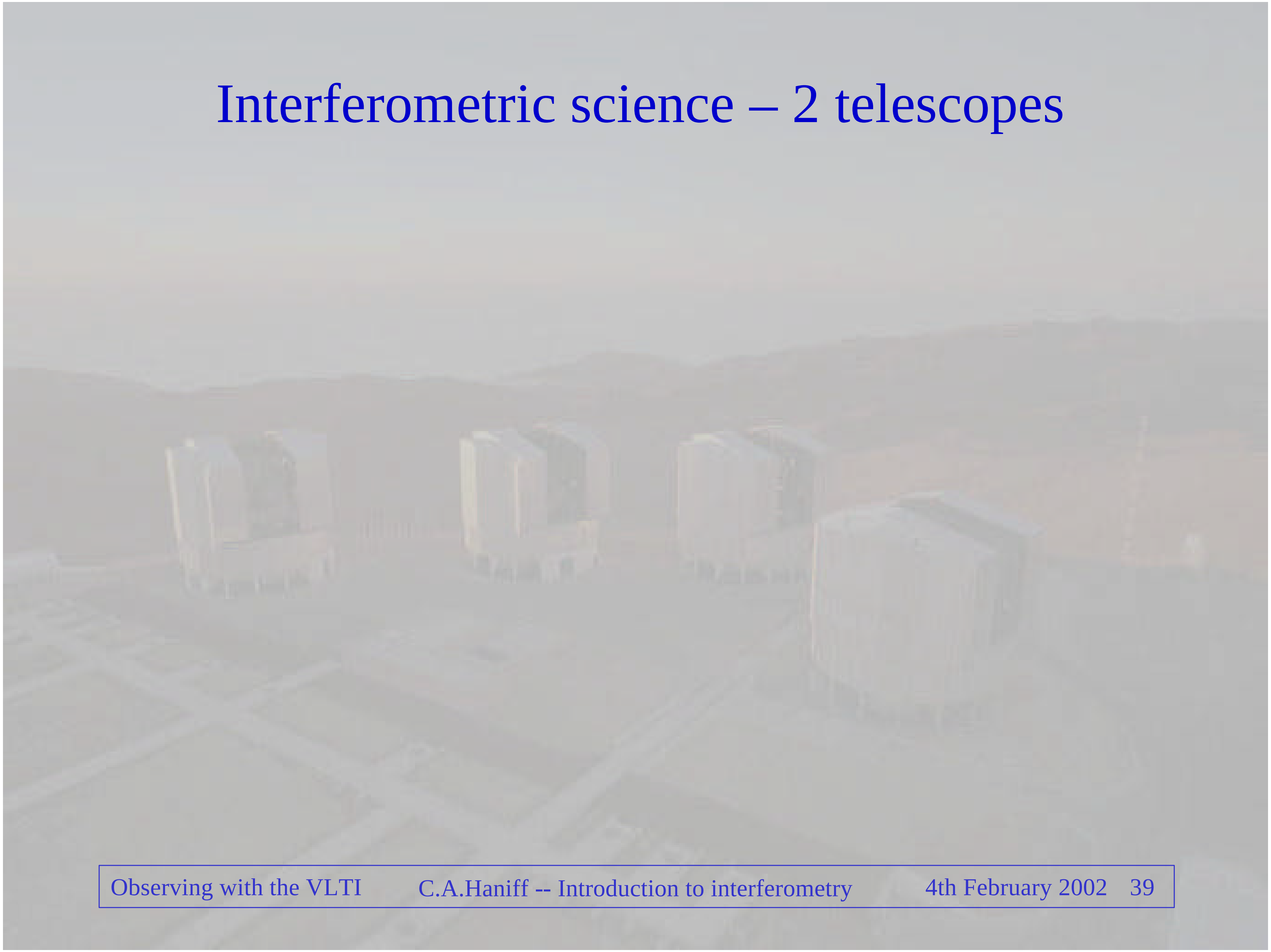
The "source" has to be bright enough to:

- Allow stabilisation of the interferometric path lengths in real time.
- Allow a reasonable signal-to-noise for the fringe parameters to be build up over some total convenient integration time. This will be measured in minutes.

Once this achieved, the faintest features in the interferometric map will be governed by the dynamic range achievable:

This in turn depends on the S/N and number of visibility data.

Interferometric science – 2 telescopes



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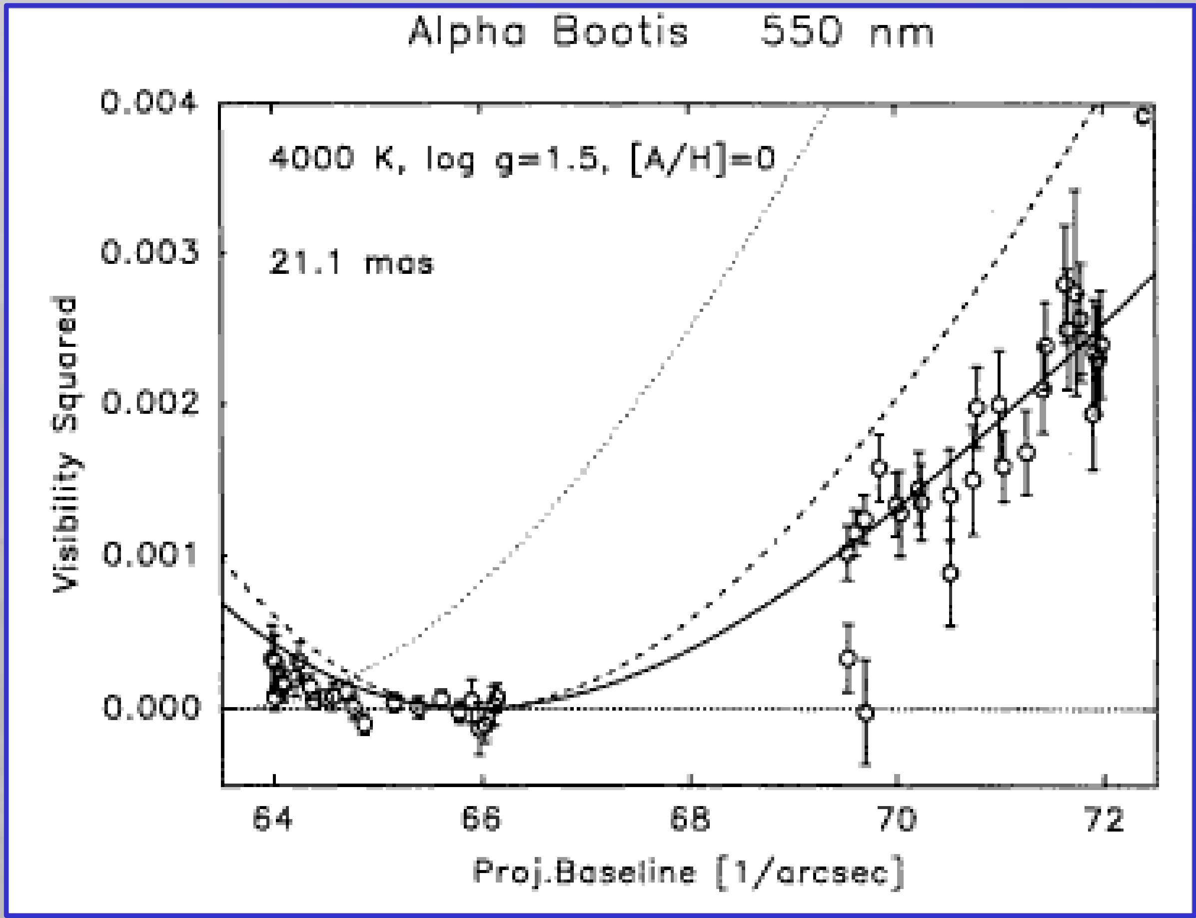
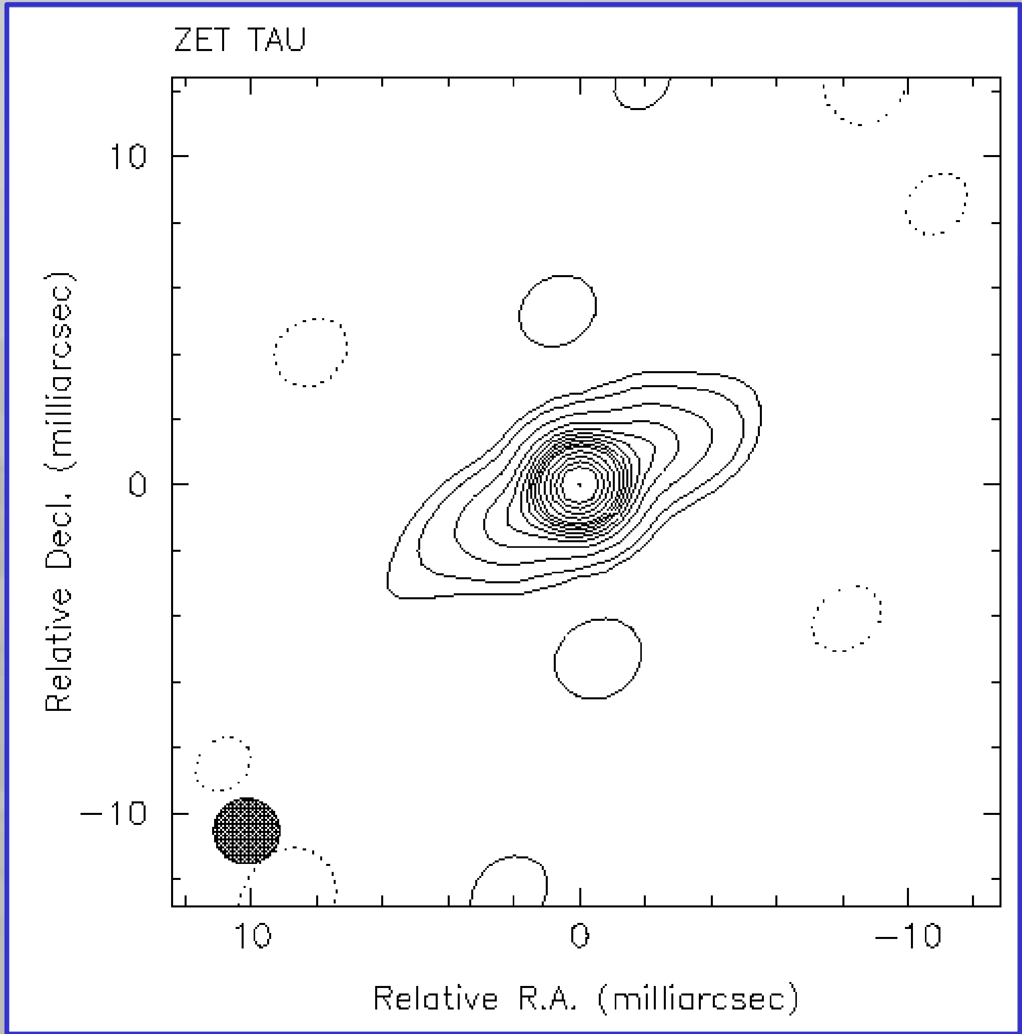
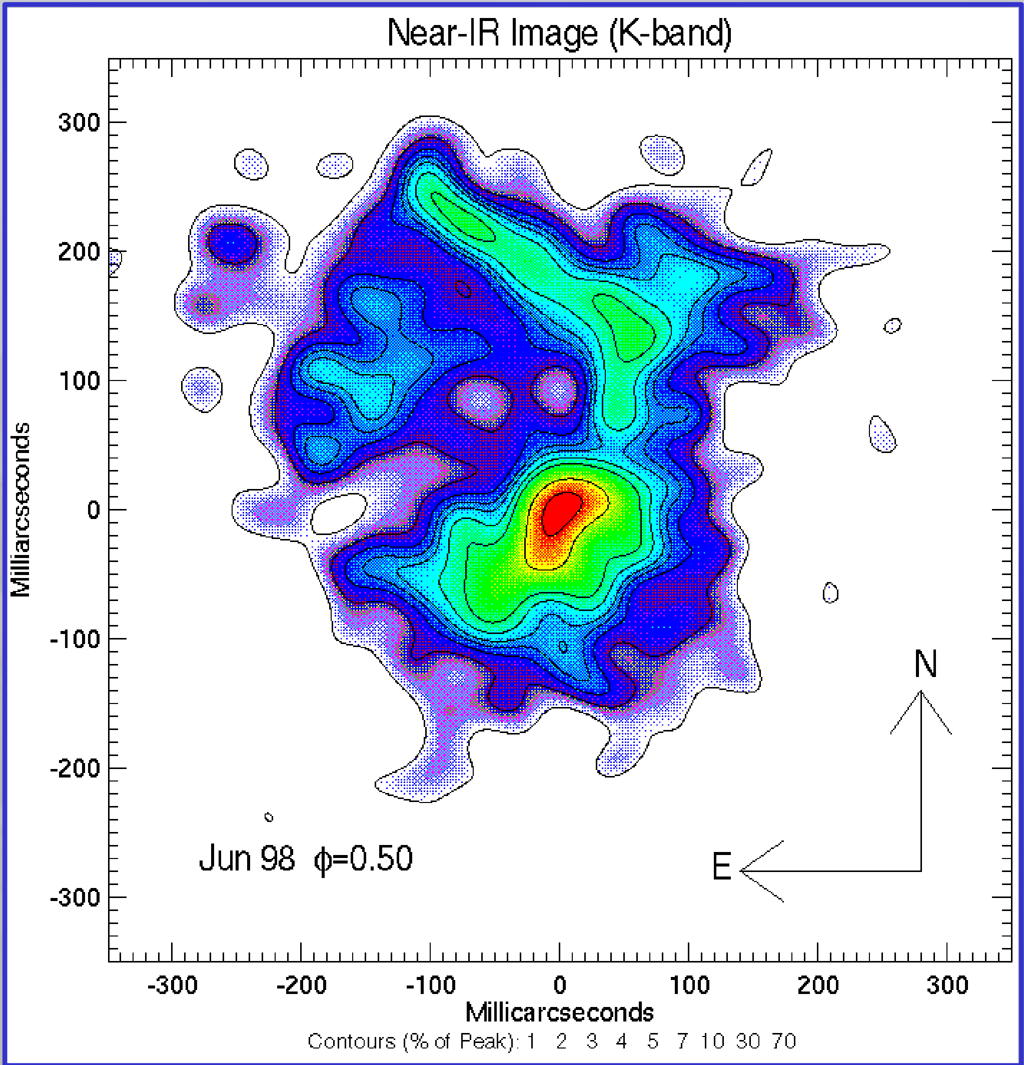


Interferometric science – 5 telescopes



Interferometric science – 21 telescopes



Interferometric science resume

- 2 telescopes: simple parametric model fitting.
- 5 telescopes: rudimentary imaging of astronomical sources.
- 21 telescopes: imaging of complex astrophysical phenomena.

Summary

- Image formation with conventional telescopes:
  - Fourier decomposition, spatial frequencies, physical baselines.
- Coherence functions:
  - Spatial & temporal: these embody the spatial & spectral content of the source.
  - Fundamental relationships are Fourier transforms.
- Interferometric measurements:
  - Fringe amplitude and phase are what is important.
  - Ability to measure these depends on signal strength & fringe modulation.
- Imaging with interferometers:
  - Rules of thumb and differences with respect to what we are used to.
  - Expectations based on 50 years of radio/optical/infrared experience.