



# Optical interferometry in practice

**EuroWinter School**

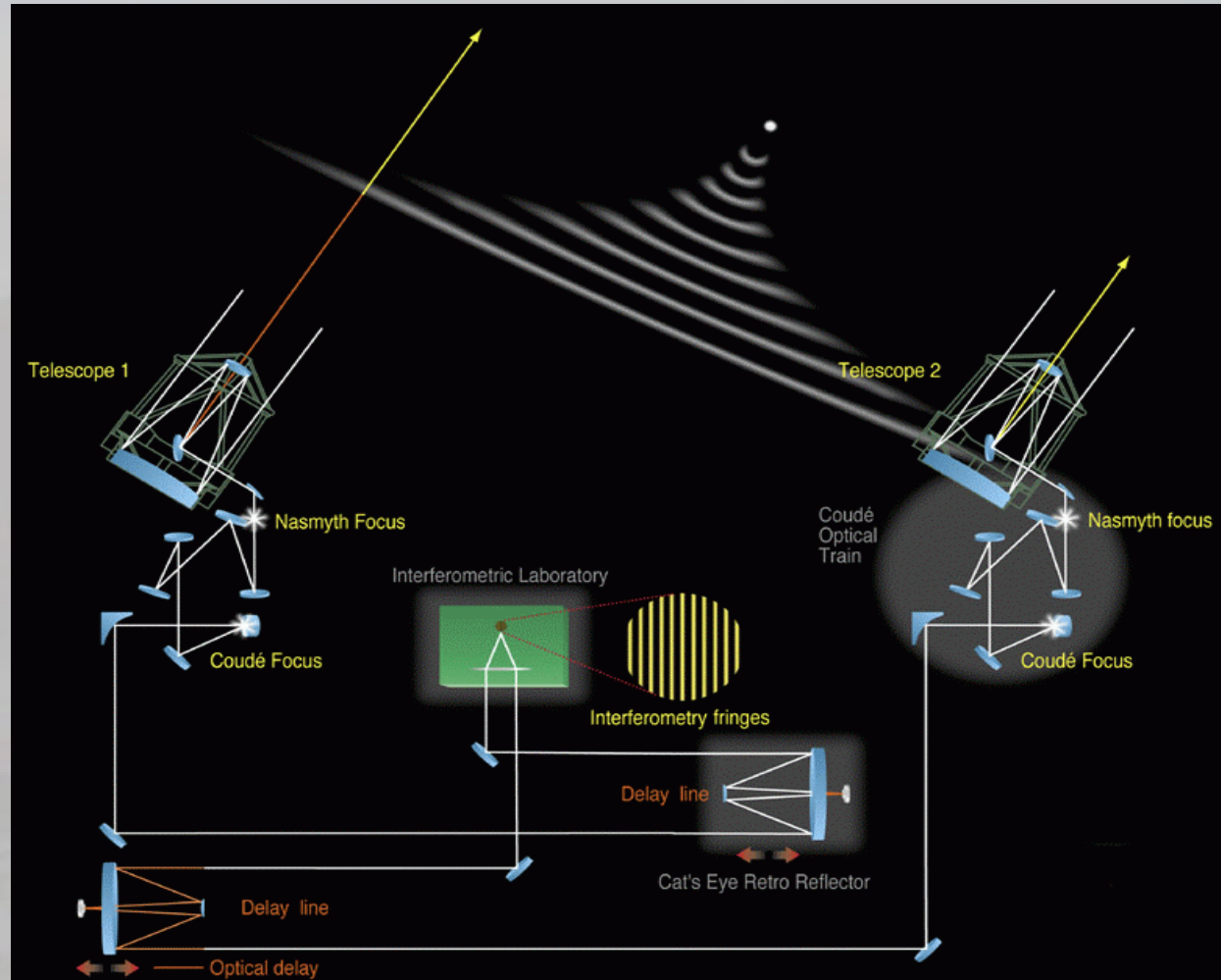
*Observing with the Very Large Telescope Interferometer*

**Les Houches, France**  
**February 3-8, 2002**

C. A. Haniff  
Cavendish Astrophysics Group  
University of Cambridge, UK  
5th February 2002

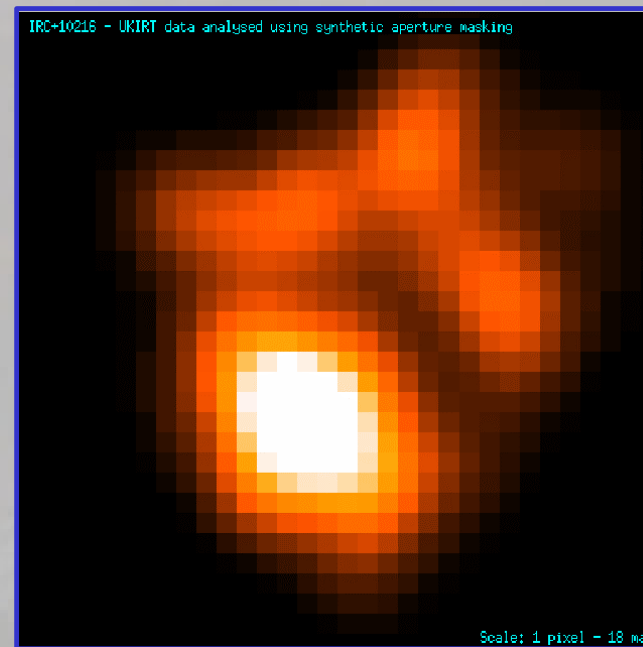
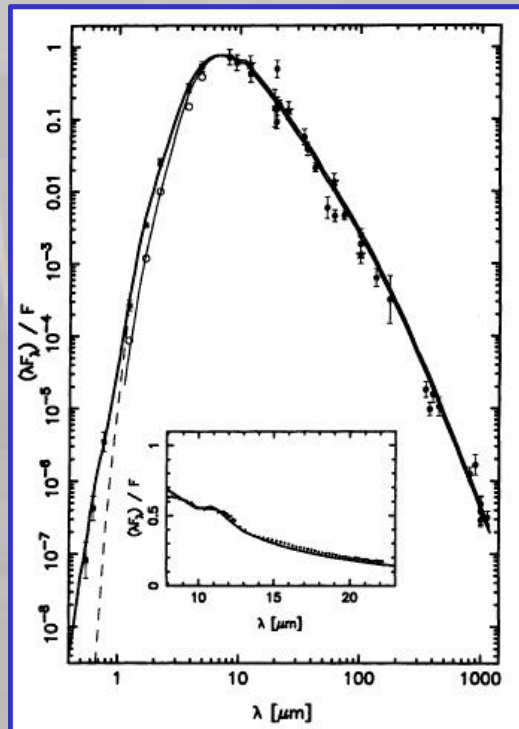
## A reminder

- **Telescopes** sample the fields at  $r_1$  and  $r_2$ .
- **Optical train** delivers the radiation to a laboratory.
- **Delay lines** assure that we measure when  $t_1=t_2$ .
- The **instruments** mix the beams and detect the fringes.



# Basic approach and rationale

- Measure visibility function on range of different baselines, each sensitive to structure on an angular scale  $\lambda/B$ .
- Interferometry is often the only way to investigate these scales.



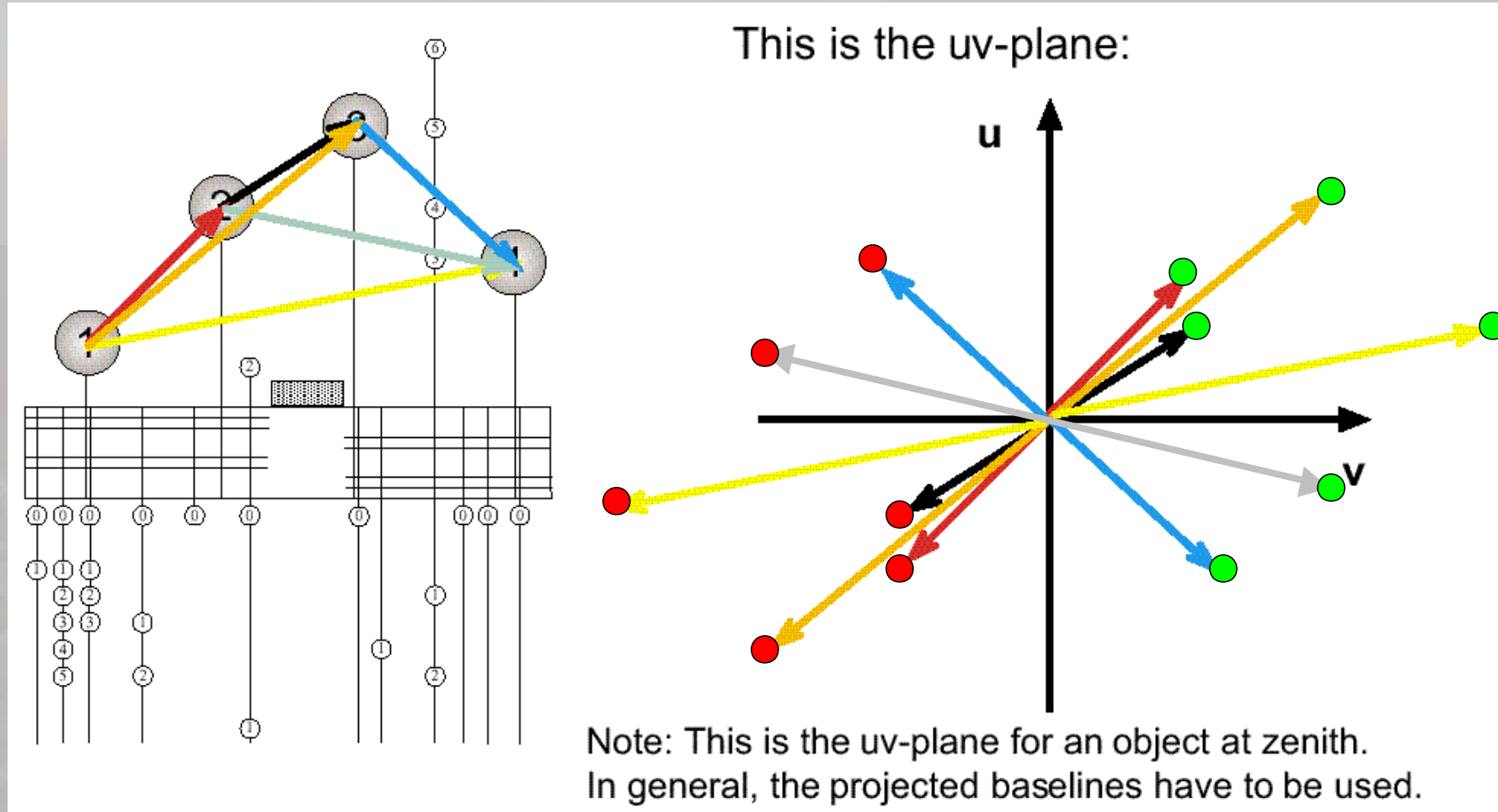
# Outline

- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - Beam combination
  - Spatial wavefront fluctuations
  - Temporal wavefront fluctuations
  - Sensitivity
  - Calibration

# Outline

- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - Beam combination
  - Spatial wavefront fluctuations
  - Temporal wavefront fluctuations
  - Sensitivity
  - Calibration

# Fourier plane sampling



## Fourier plane sampling (cont.)

In practice rather than re-locate the telescopes to measure different spatial frequencies, we take advantage of the Earth's rotation. In this case the tips of the  $uv$  vectors sweep out **ellipses**.

The properties of these will be governed by:

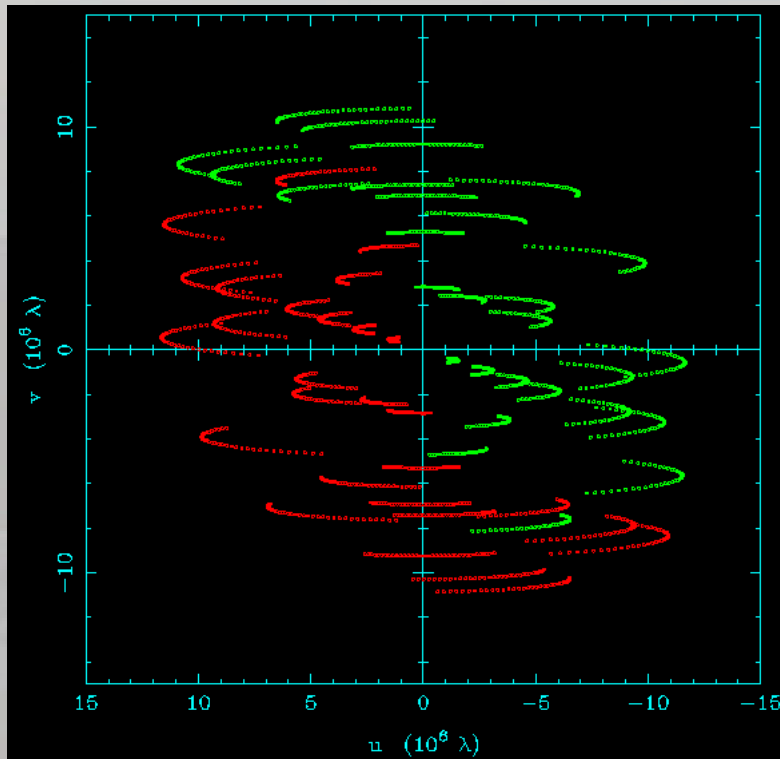
- The hour angles of the observation.
- The declination of the source.
- The stations being used.

Issues to be thinking about will include:

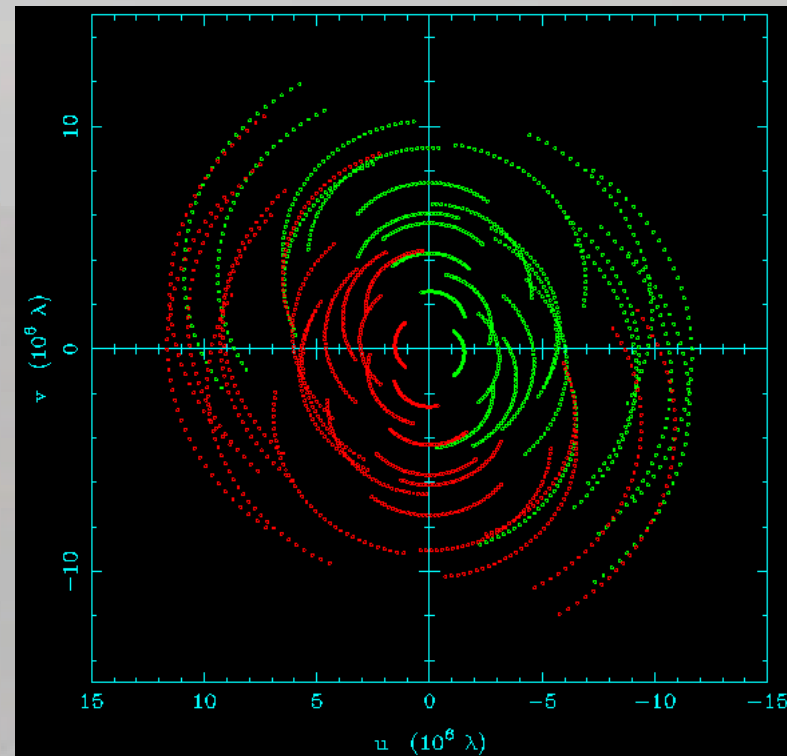
- Is there any shadowing of the telescopes by each other?
- The allowed range of the delay lines - are they long enough?
- The zenith distance - will the seeing be too poor at low elevations?
- Can the interferometer **fringe-track** ok?

# Examples of Fourier plane coverage

Dec -15



Dec -65

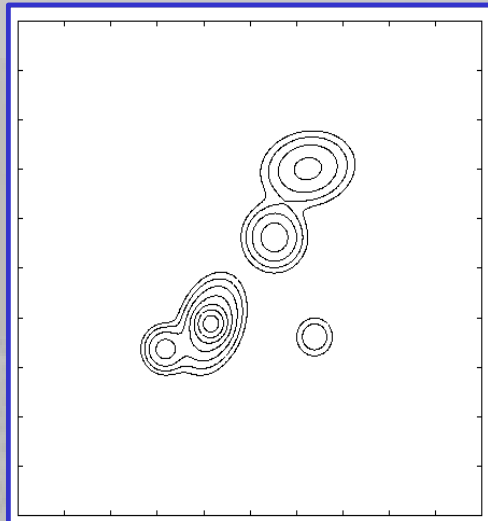


Whatever these look like, don't forget the “rules of thumb”!

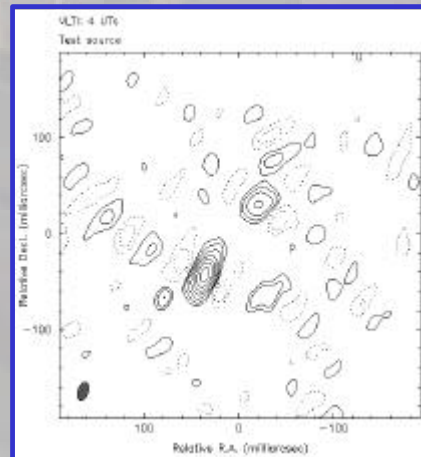
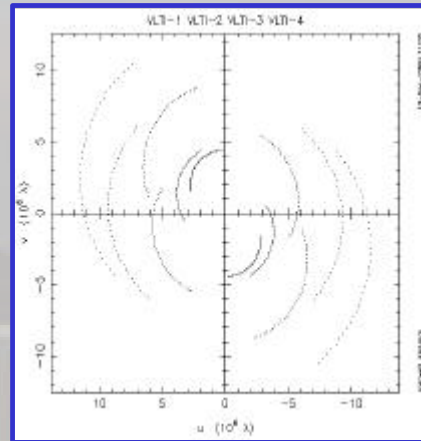


# Image complexity and number of telescopes

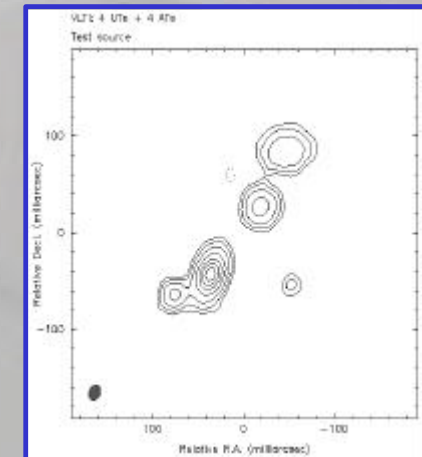
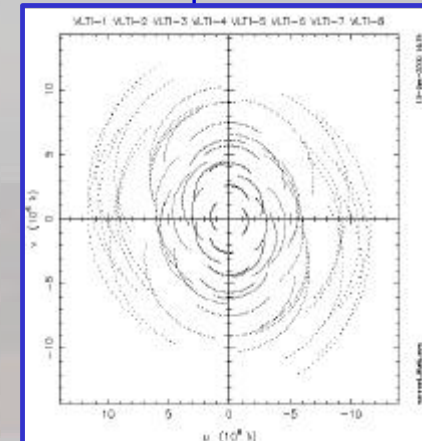
Model



4 telescopes, 6 hours



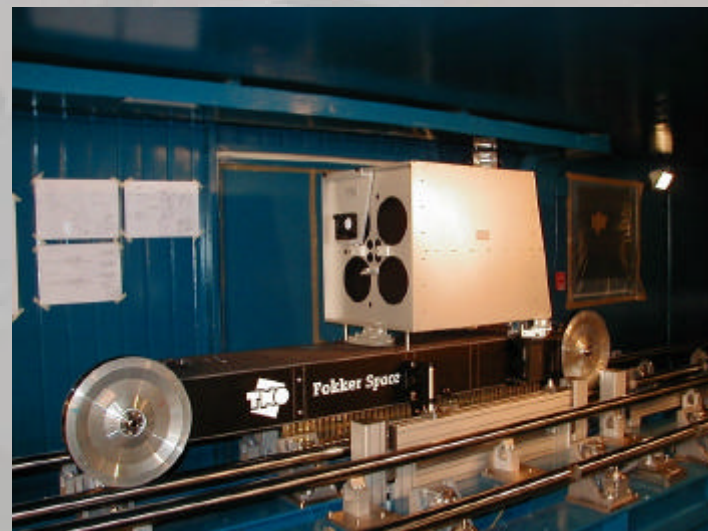
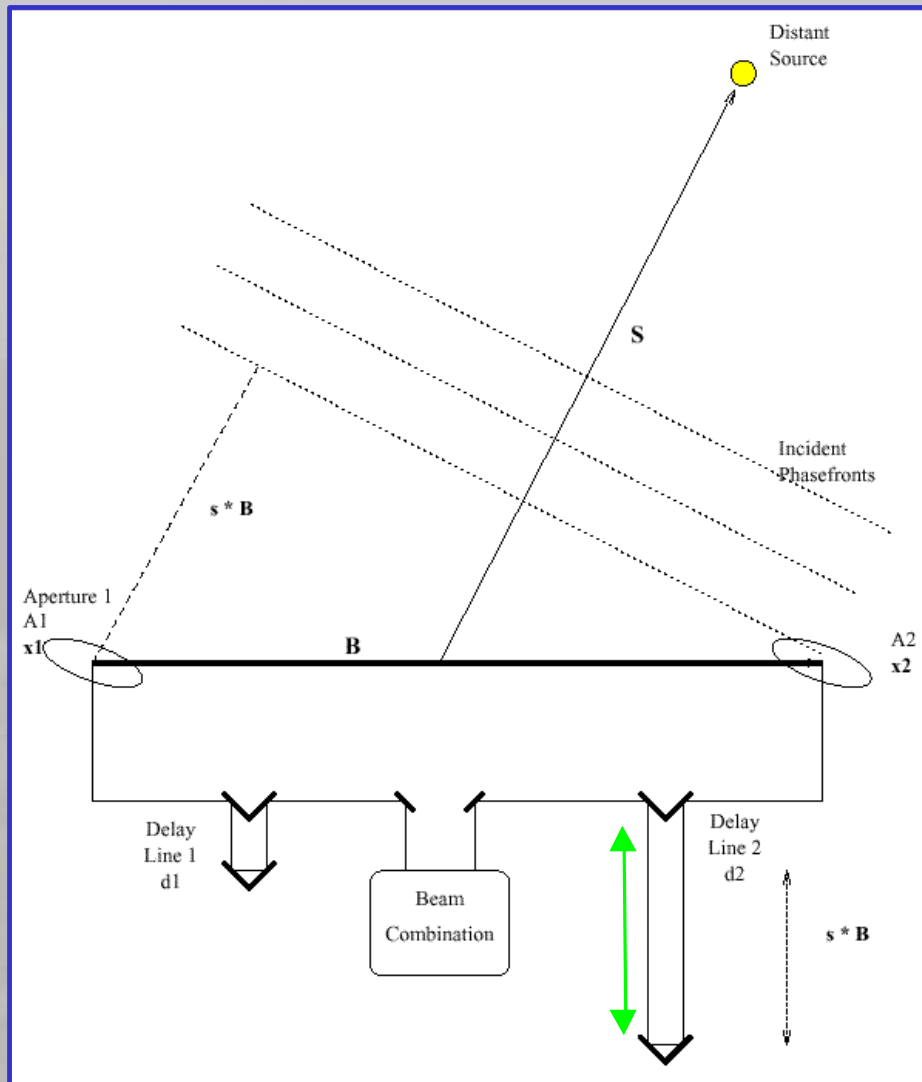
8 telescopes, 6 hours



# Outline

- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - **Delay lines**
  - Beam combination
  - Spatial wavefront fluctuations
  - Temporal wavefront fluctuations
  - Sensitivity
  - Calibration

# Delay lines



## Delay lines (ii)

- When the source is at the zenith no delay compensation is needed

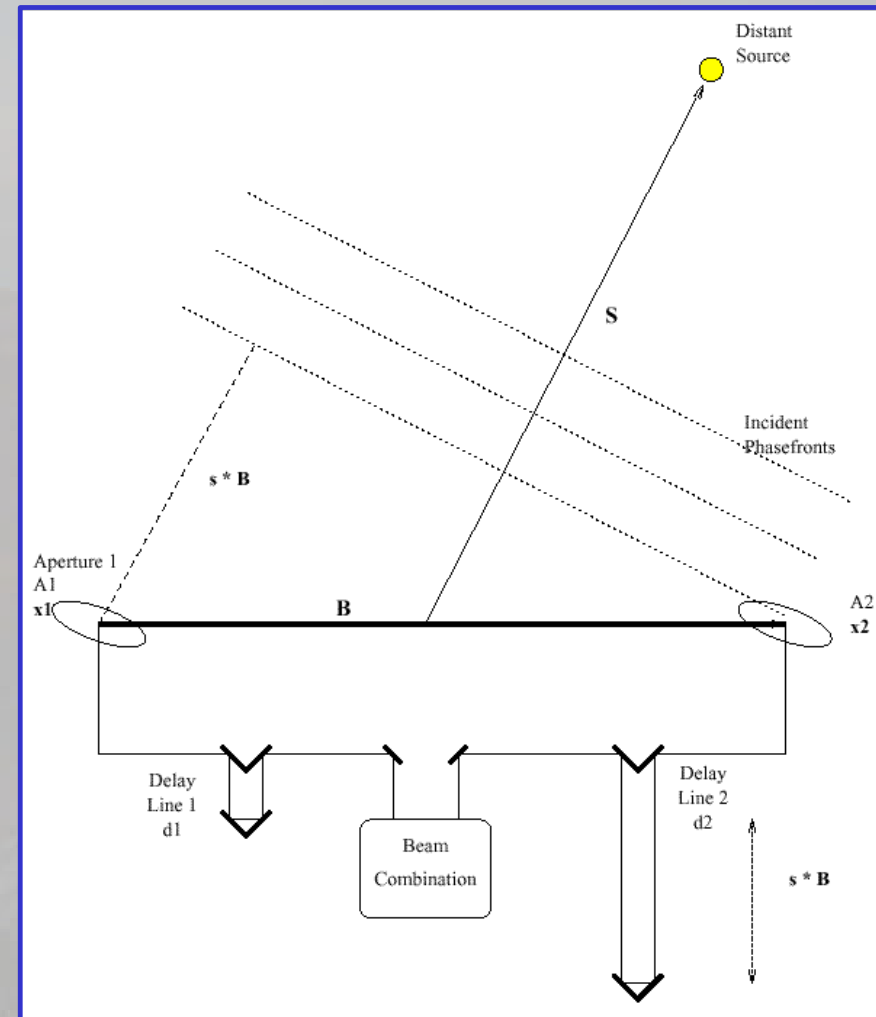
VLTI has  $\text{opd}_{\text{max}} \sim 120\text{m}$ .

- The OPD correction varies as  $B \cos(\theta) d\theta/dt$ , with  $\theta$  the zenith angle.

VLTI has  $v_{\text{max}} \sim 0.5\text{m/s}$ .

- The correction has to be better than  $l_{\text{coh}} \sim \lambda^2/\Delta\lambda$ .

VLTI stability is  $\leq 14\text{nm rms}$ .



## Some practical caveats

- Unless very specialized beam-combining optics are used it is only possible to correct the OPD for a single direction in the sky.
  - This is what gives rise to the FOV limitation:  $\theta_{\max} \leq [\lambda/B][\lambda/\Delta\lambda]$ .
- As the optical train is in air, the OPD is actually different for different wavelengths since the refractive index  $n = n(\lambda)$ .
  - This **longitudinal dispersion** implies that different locations of the delay line carts will be required to equalize the OPD at different wavelengths!
  - For a 100m baseline and a source  $50^\circ$  from the zenith this  $\Delta\text{OPD}$  corresponds to  $\sim 10\mu\text{m}$  between 2.0-2.5 $\mu\text{m}$ .
  - More precisely, this implies the use of a **spectral resolution**,  $R > 5$  (12) to ensure good fringe contrast ( $>90\%$ ) in the K (J) band.

# Outline

- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - **Beam combination**
  - Spatial wavefront fluctuations
  - Temporal wavefront fluctuations
  - Sensitivity
  - Calibration

# Beam combination

The essential principle here is:

- Add the E fields,  $E_1 + E_2$ , and then detect the time averaged intensity:

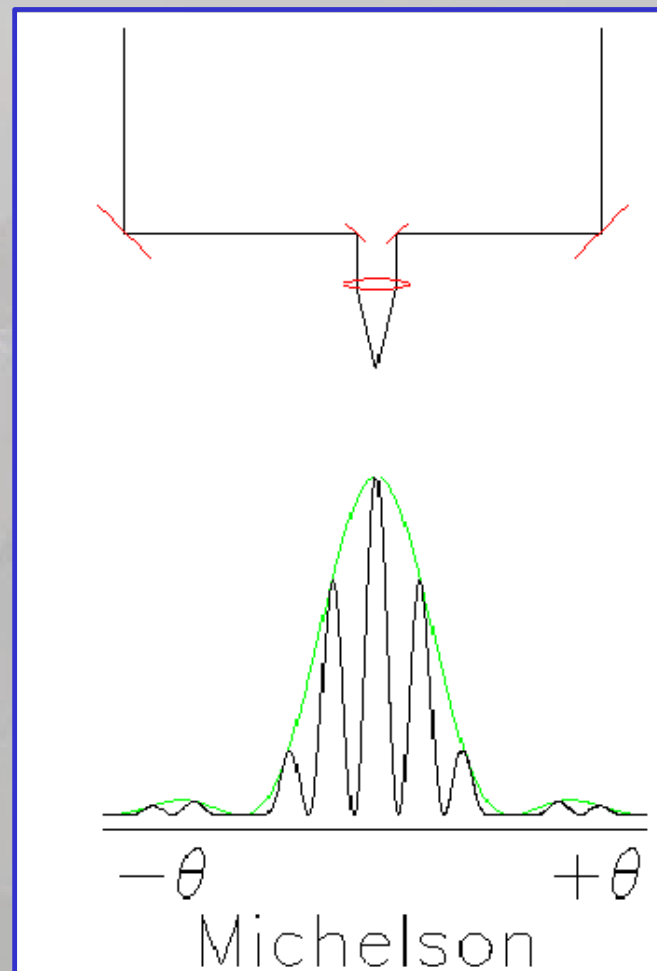
$$\begin{aligned}\dot{a}(E_1 + E_2) \times (E_1 + E_2)^* \tilde{n} &= \dot{a}|E_1|^2 \tilde{n} + \dot{a}|E_2|^2 \tilde{n} + \dot{a}E_1 E_2^* \tilde{n} + \dot{a}E_2 E_1^* \tilde{n} \\ &= \dot{a}|E_1|^2 \tilde{n} + \dot{a}|E_2|^2 \tilde{n} + \dot{a}2|E_1||E_2| \cos(\varphi) \tilde{n}\end{aligned}$$

where  $\varphi$  is the phase difference between  $E_1$  and  $E_2$ .

- In practice there are two **straightforward** ways of doing this:
  - Image plane combination:
    - AMBER and aperture masking experiments.
  - Pupil plane combination:
    - MIDI and systems using fibre couplers (VINCI).

# Image plane combination

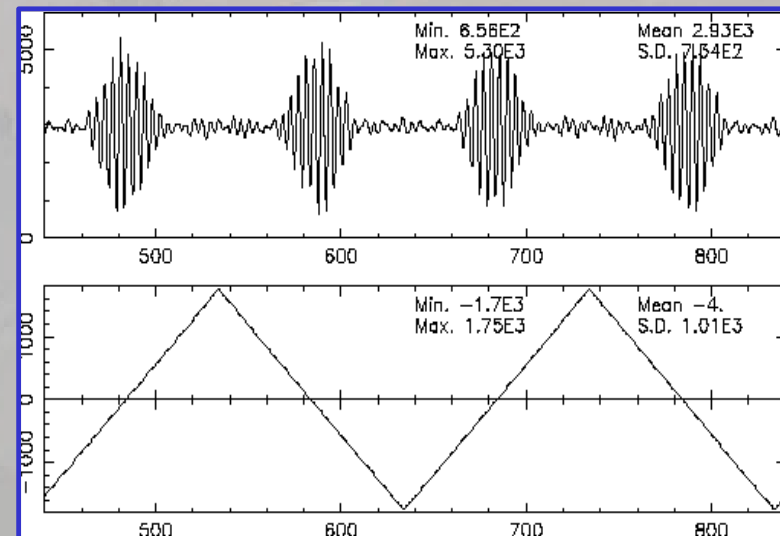
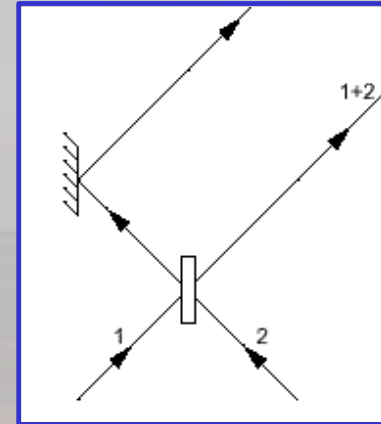
- Mix the signals in a **focal plane** as in a Young's slit experiment:
- In the focused image the transverse co-ordinate measures the delay.
- Fringes encoded by use of a non-redundant input pupil.
- The choice of the number of beams combined is selected to optimise the signal-to-noise.
- Possible to use dispersion prior to detection in the direction perpendicular to the fringes. Allows measurement of coherence function at multiple  $\lambda$ .





# Pupil plane combination

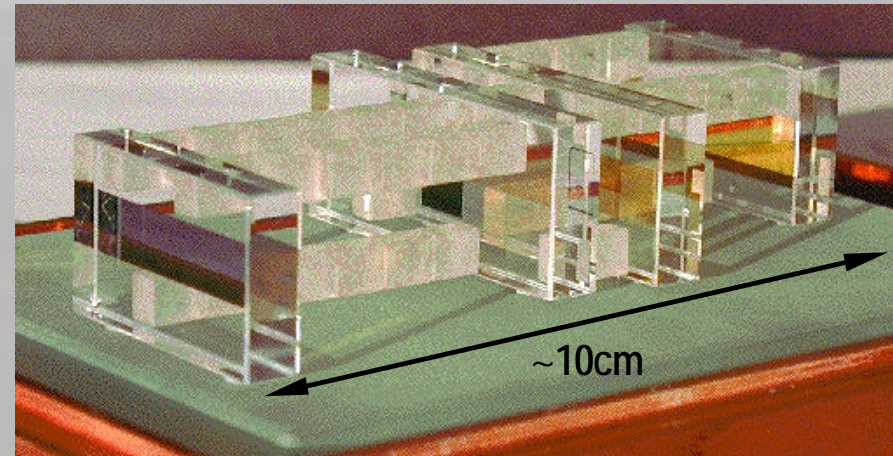
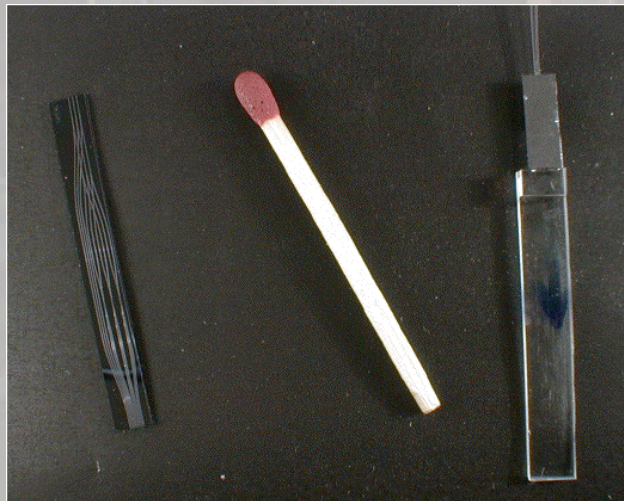
- Mix the signals by superposing **afocal** beams:
- Then focus superposed beams onto a single element detector.
- Fringes encoded by use of a non-redundant modulation of delay of each beam.
- Fringes are visualised by measuring intensity versus time.
- The number of beams combined is selected to optimise the S/N and spectral dispersion can be used prior to detection.



## Issues for the future

- Stability and throughput.
- Spectral bandpass.
- Ability to deal with large number of input beams.

Integrated optics 2 and 3-way combiners



Bulk optics 4-way 1-2.5 $\mu\text{m}$  combiner.

# Outline

- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - Beam combination
  - **Spatial wavefront fluctuations**
  - Temporal wavefront fluctuations
  - Sensitivity
  - Calibration

# Spatial fluctuations

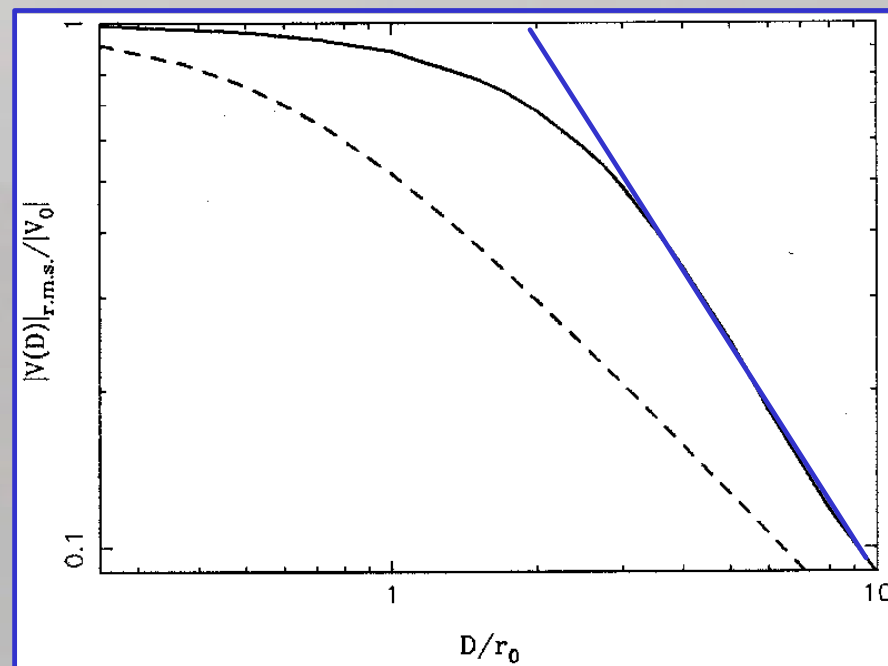
- These are characterized by Fried's parameter,  $r_0$ .
  - The circular aperture size over which the mean square wavefront error is approximately 1 radian<sup>2</sup>.
  - This scales as  $\lambda^{6/5}$ .
  - The fluctuations exhibit a particularly steep spectrum:  $\propto \kappa^{-11/3}$ .
  - And are potentially limited by an **outer scale**,  $L_0$ , beyond which their strength saturates.
  - Tel. Diameters  $>$  or  $<$   $r_0$  delimit different regimes of **instantaneous** image structure:
    - $D < r_0 \Rightarrow$  quasi-diffraction limited images with image motion.
    - $D > r_0 \Rightarrow$  high contrast speckled (distorted) images.
  - Median  $r_0$  value at Paranal is **15cm** at  $0.5\mu\text{m}$ .

# Impact on interferometry

- How does this impinge on interferometry?

- Reduces the rms visibility (-----) amplitude as  $D/r_0$  increases.
- Leads to increased fluctuations in  $V$ .
- Both the above  $\Rightarrow$  **loss** in sensitivity.
- Impacts on reliability of calibration.

- **Moderate** improvement is possible with tip-tilt correction ( ——— ).



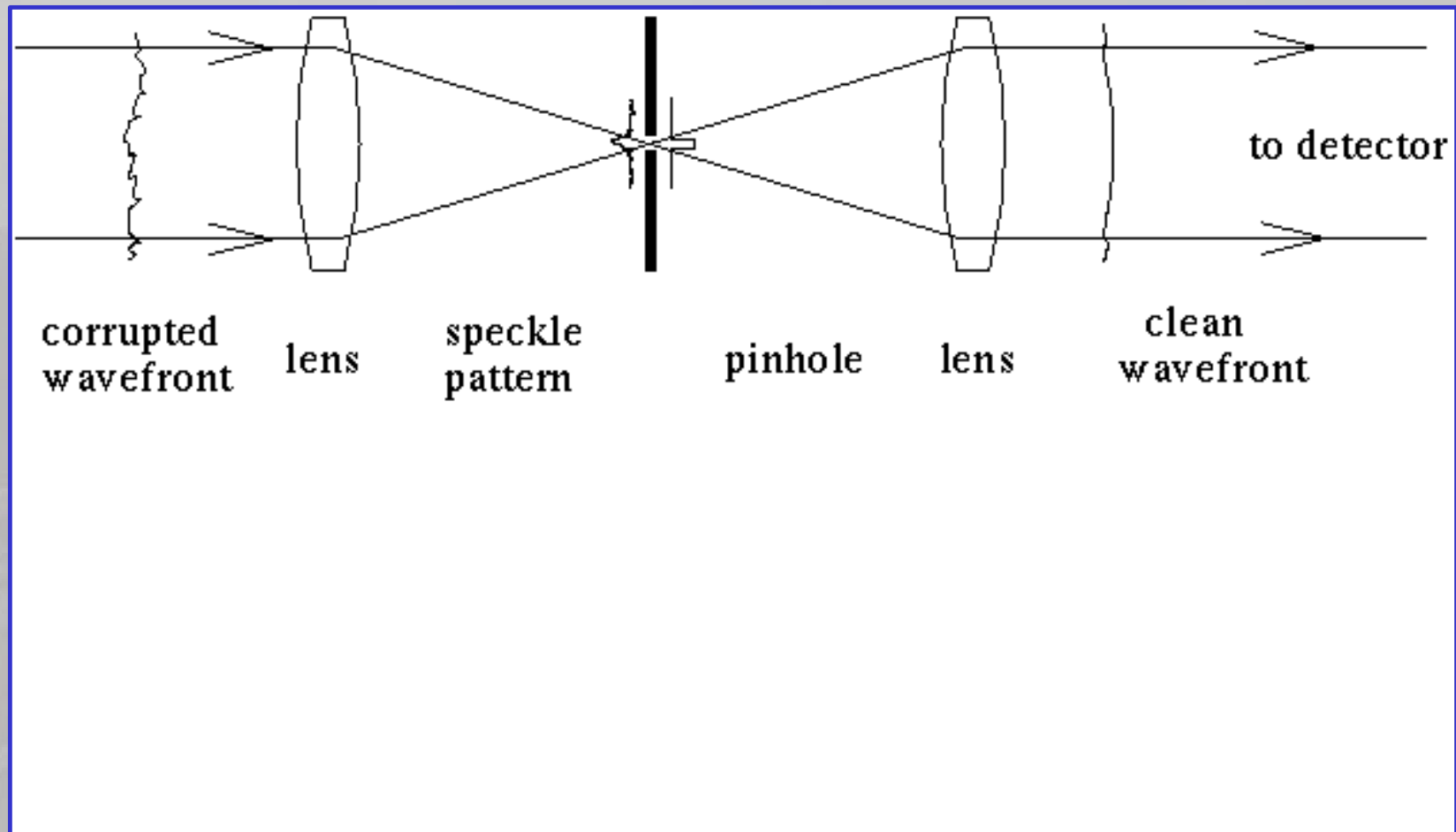
# Solutions

In principle, there are two approaches to deal with spatial fluctuations for telescopes of finite size:

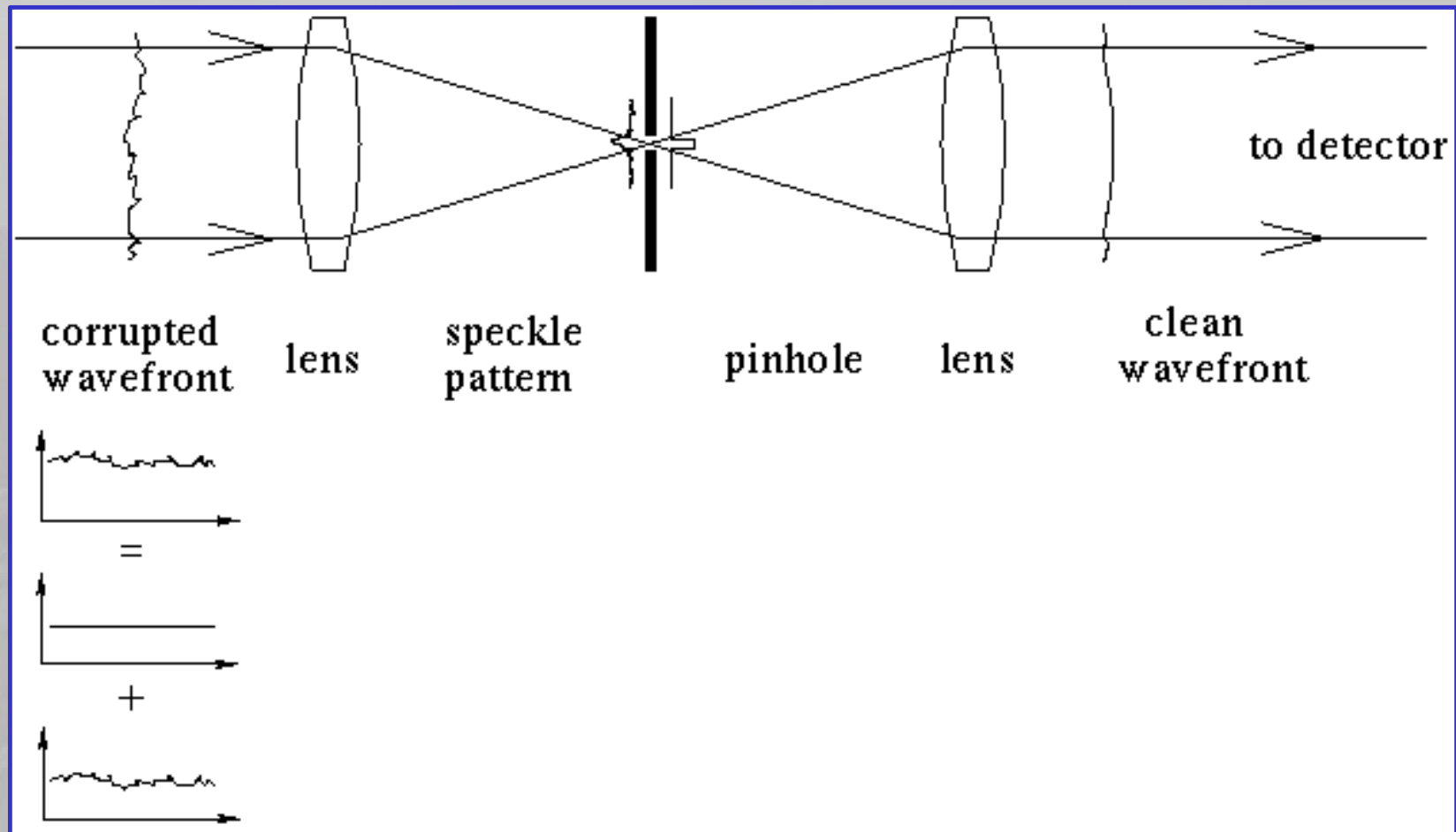
l/mm	1.25	1.65	2.2	3.5	5.0
ATs	3.8	2.7	1.9	1.1	0.7
UTs	16.7	11.9	8.4	4.8	3.1

- Use an **adaptive optics** system correcting higher order Zernike modes:
  - Can use either the source or an off-axis reference star to sense atmosphere.
  - But need to worry about how bright and how far off axis is sensible.
- Instead, **spatially filter** the light arriving from the collectors:
  - This trades off a fluctuating visibility for a variable throughput.
  - Can use either a monomode optical fibre or a pinhole.

# Spatial filtering

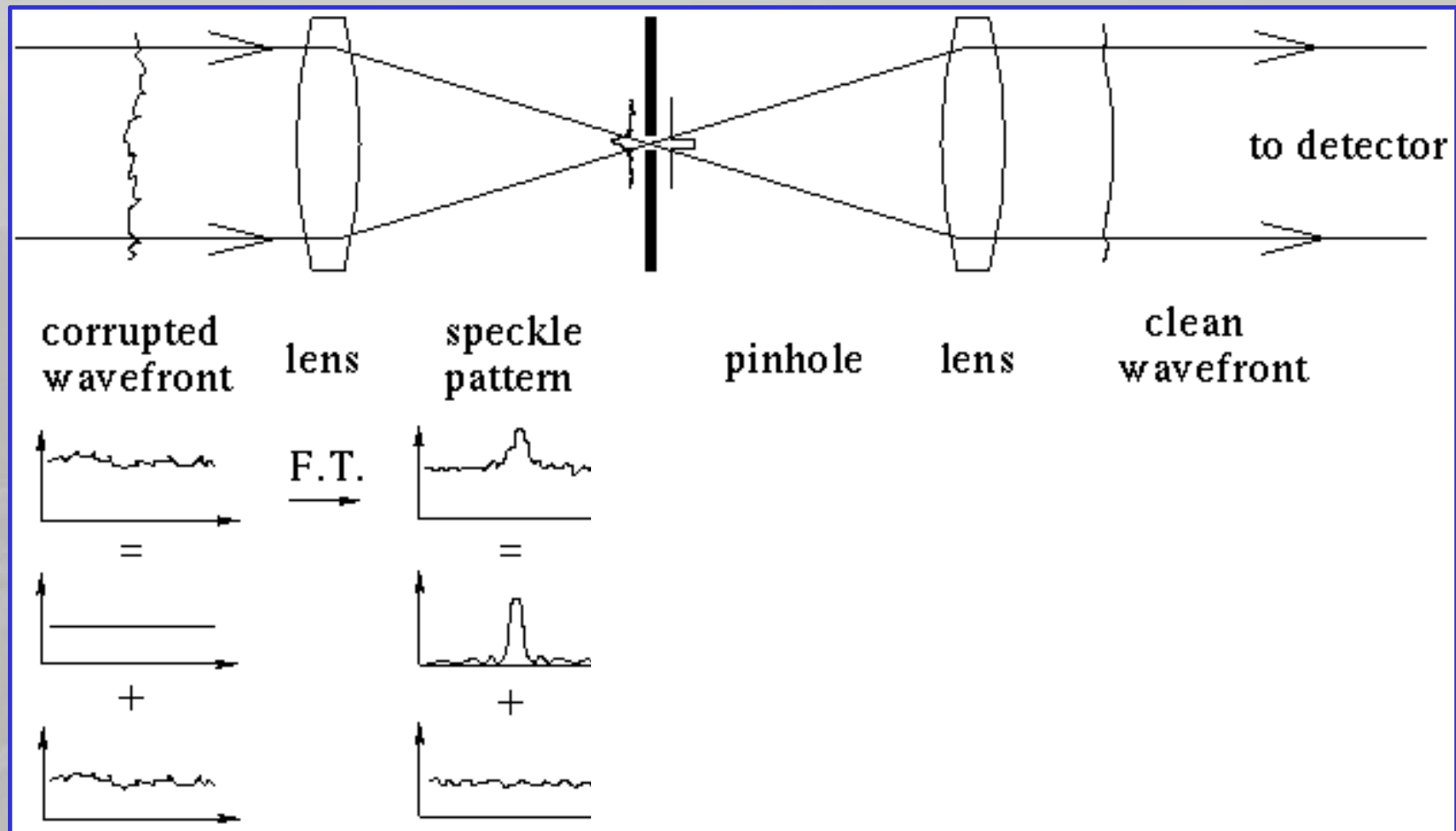


# Spatial filtering

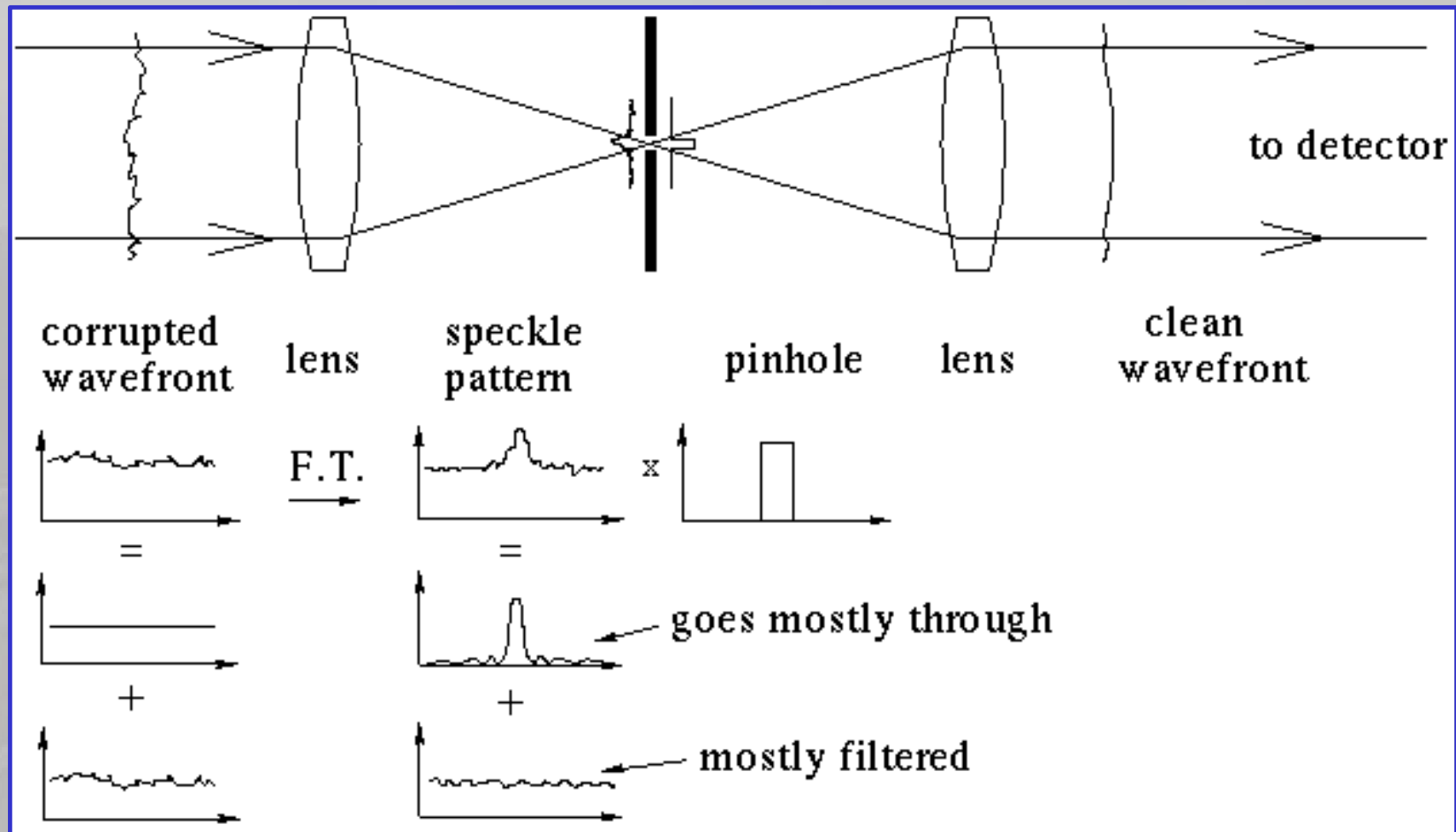




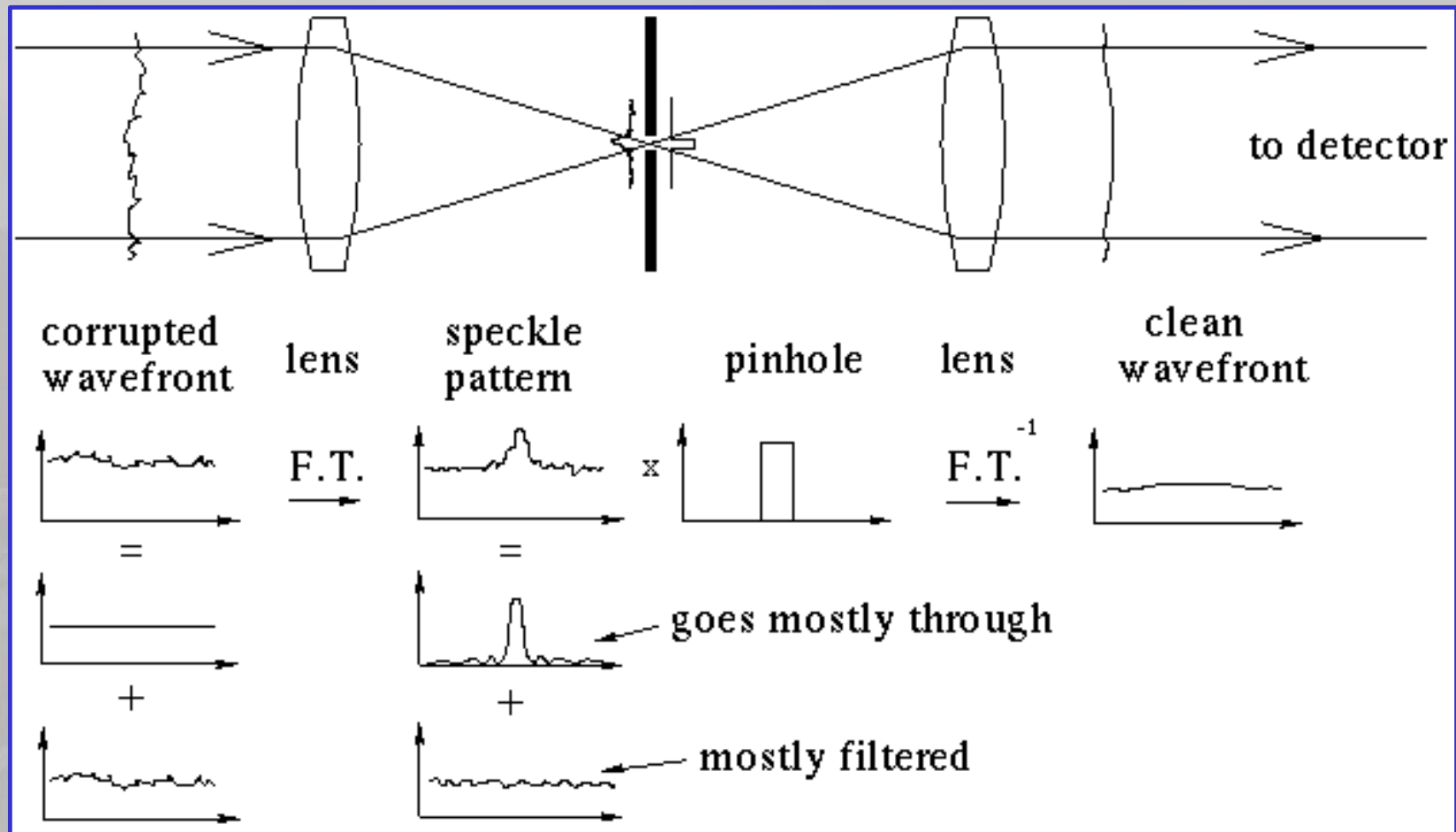
# Spatial filtering



# Spatial filtering

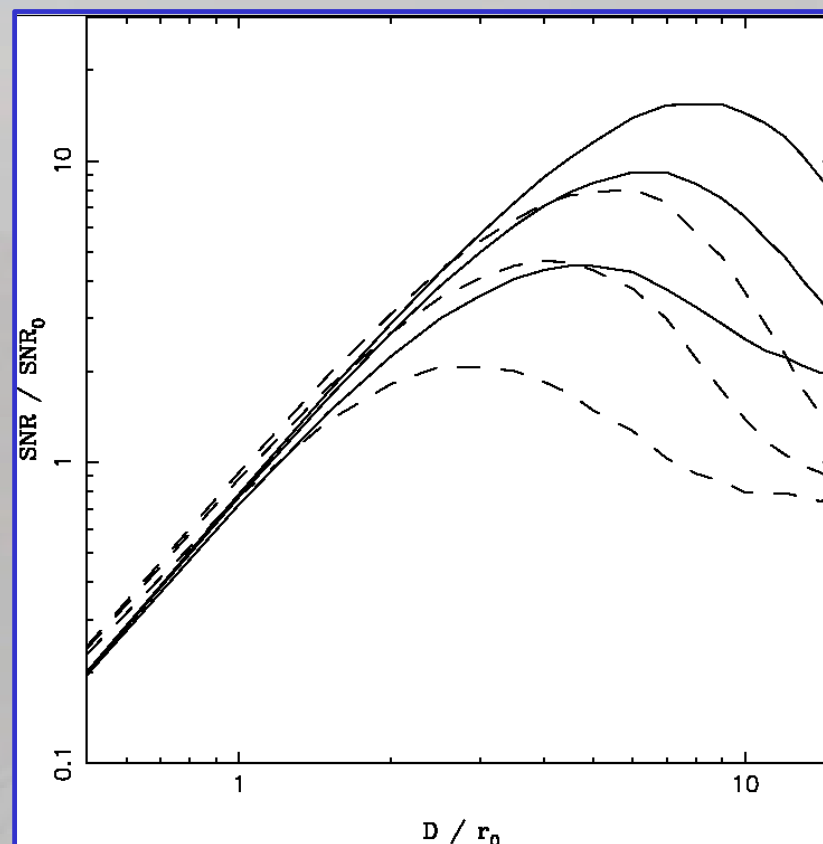


# Spatial filtering

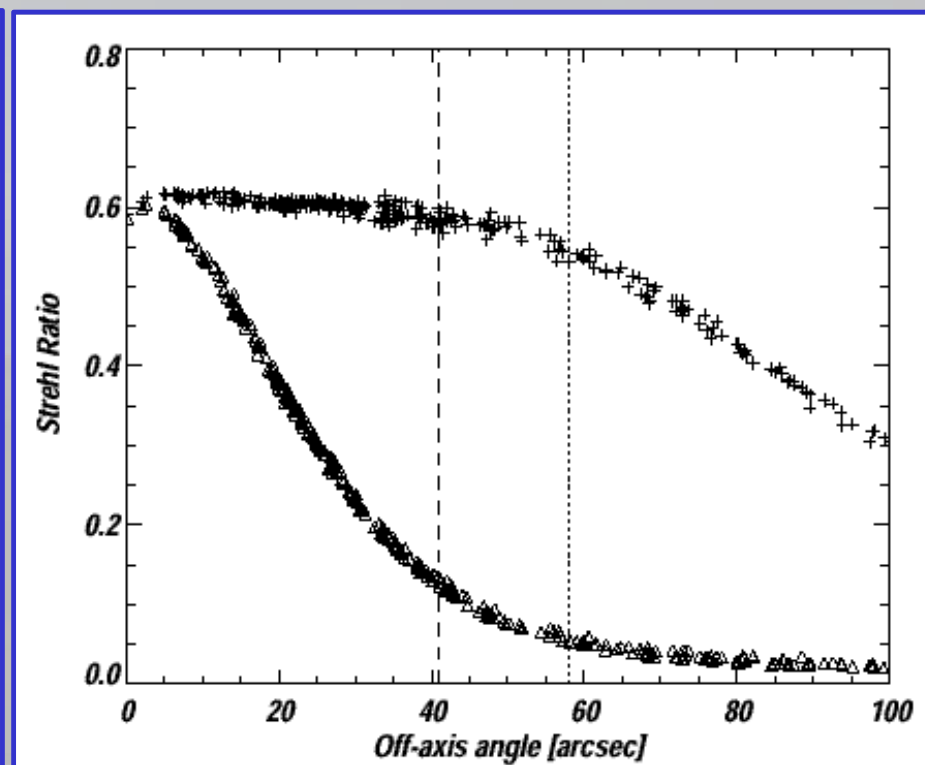
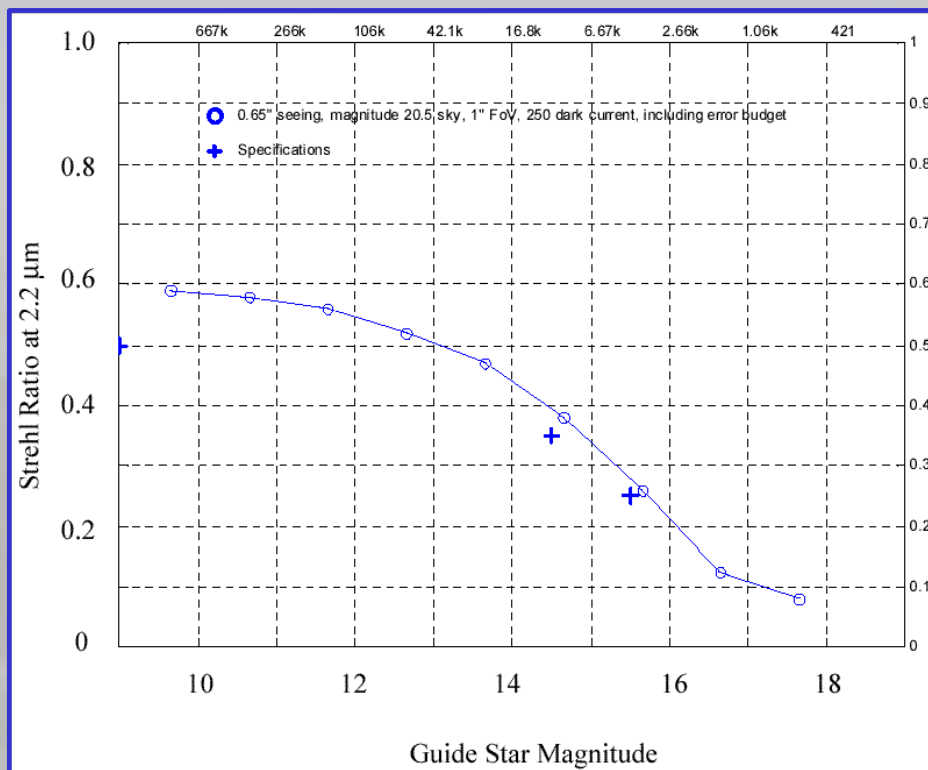


## Mixed and optimal strategies

- What is actually of interest here is using larger apertures to get better sensitivity to see **fainter** sources:
  - Curves show fringe power S/N with and without the use of a spatial filter.
  - And with 2, 5 and 9 non-piston Zernike modes corrected by an AO system.
- Implications are:
  - Spatial filtering always helps.
  - Can work with large  $D/r_0$  (e.g.  $\leq 10$ ).
  - For perfect wavefronts  $S/N \propto D$ .



# Spatial fluctuations and AO



- Influence of guide-star magnitude. This is for MACAO at the VLTI.
- Influence of off-axis angle. This is for a generic 8m telescope at M. Kea.

NGS systems basically offer only a modest improvement in sky coverage, but are vital in allowing photons to be collected faster for bright sources.

# Outline

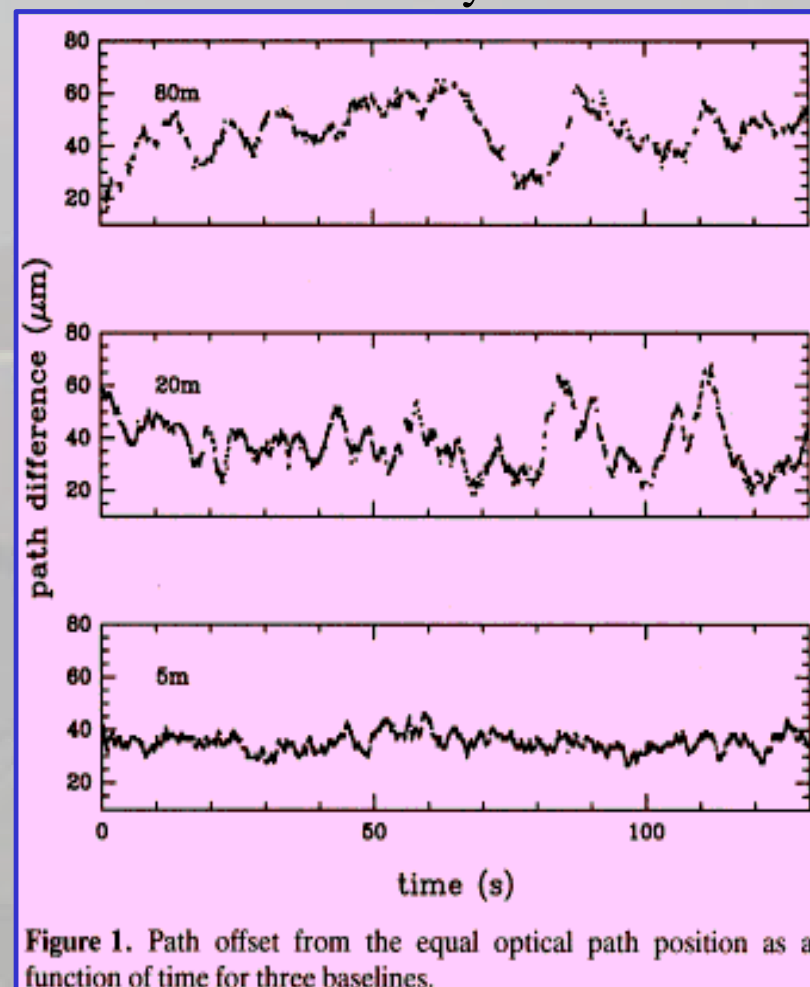
- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - Beam combination
  - Spatial wavefront fluctuations
  - **Temporal wavefront fluctuations**
  - Sensitivity
  - Calibration

# Temporal fluctuations

- These are characterized by a coherence time,  $t_0$ .
  - Heuristically this is the time over which the wavefront phase changes by approximately 1 radian.
- Related to spatial scale of turbulence and windspeed:
  - Assume that Taylor's "frozen turbulence" hypothesis holds, i.e. that the timescale for evolution of the wavefronts is long compared with the time to blow past your telescope.
  - Obtain a characteristic timescale  $t_0 = 0.314 r_0/v$ , with  $v$  a nominal wind velocity. Scales as  $\lambda^{6/5}$ .
- Typical values can range between 3-20ms at  $0.5\mu\text{m}$ .
  - Expect larger spatial scales to correspond to longer temporal ones.
  - Some evidence that windspeed is inversely correlated with  $r_0$ .
  - Recent data from Paranal show median value of  $\sim 20\text{ms}$  at  $2.2\mu\text{m}$ .

# Impact on interferometry

- Temporal fluctuations provide a **fundamental** limit to the sensitivity of optical arrays.
  - Short-timescale fluctuations **blur** fringes:
    - Need to make **measurements** on timescales shorter than  $\sim t_0$ .
  - Long-timescale fluctuations move the fringe envelope out of measurable region.
    - Fringe envelope is few microns
    - Path fluctuations tens of microns.
    - Requires **dynamic tracking** of piston errors.





# Perturbations to the phase of $V$

- Apart from forcing any interferometric measurements to be made on a very short timescale, the other key problem introduced by temporal wavefront fluctuations is that they alter the **phase** of the measured visibility (i.e. coherence) function.

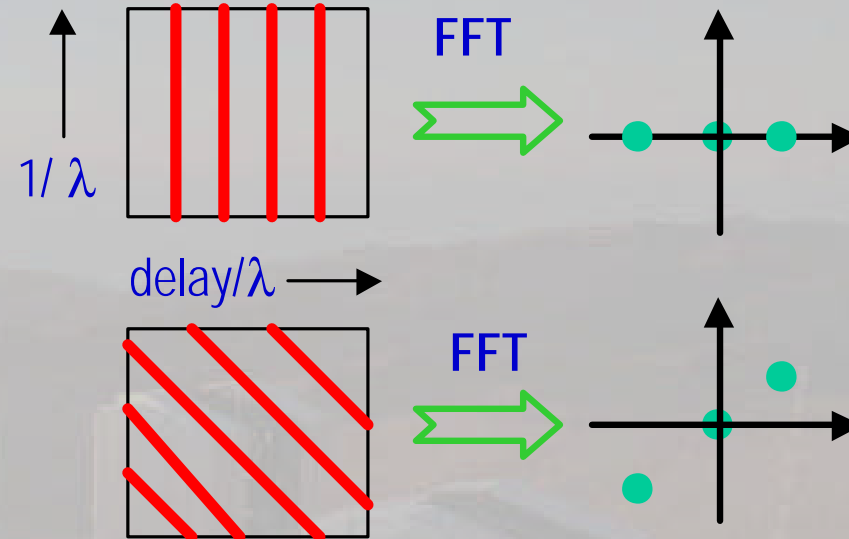
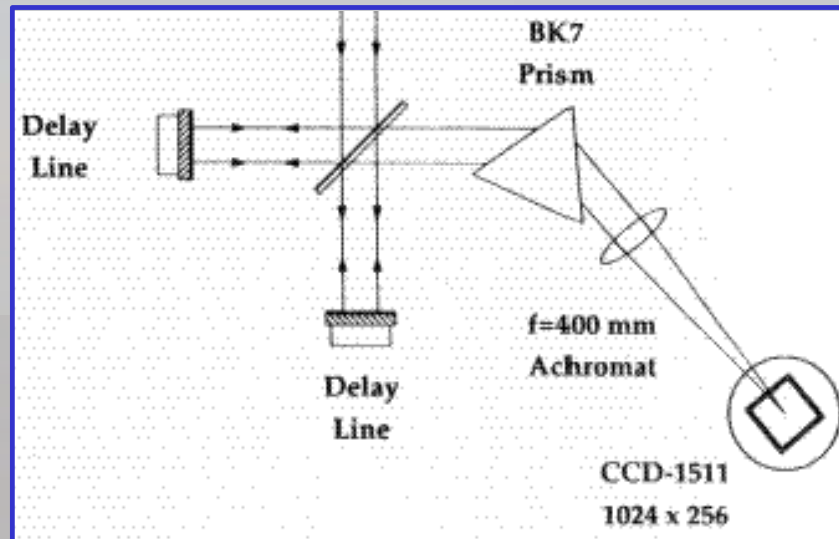
Simple Fourier inversion of the coherence function becomes impossible.

- How do we get around this problem?
  - Dynamically track the atmospheric excursions at the sub-wavelength level
    - Phase is then a useful quantity.
  - Measure something useful that is independent of the fluctuations.
    - Relative phase.
    - Closure phase.

# Fringe tracking basics

- We can identify several possible fringe-tracking systems:
  - Those that ensure we are **close** to the coherence envelope.
  - Those that ensure we remain **within** the coherence envelope.
  - Those that **lock** onto the white-light fringe.
- The first two of these still need to be combined with measurements of observables that preserve useful phase information.
- Only the last of these allows for direct Fourier inversion of the measured visibility function.
- As an aside, the second of these is generally referred to as “**envelope**” tracking or **coherencing**, while the third is often called “**phase**” tracking.

# Envelope tracking



Fringe envelope tracking methods - e.g. **group delay** tracking.

- Observe fringes in **dispersed** light.
- Dispersed fringes are tilted when OPD non-zero
- Recover fringe envelope position using 2-D power spectrum.
- Can integrate for several seconds – high sensitivity.

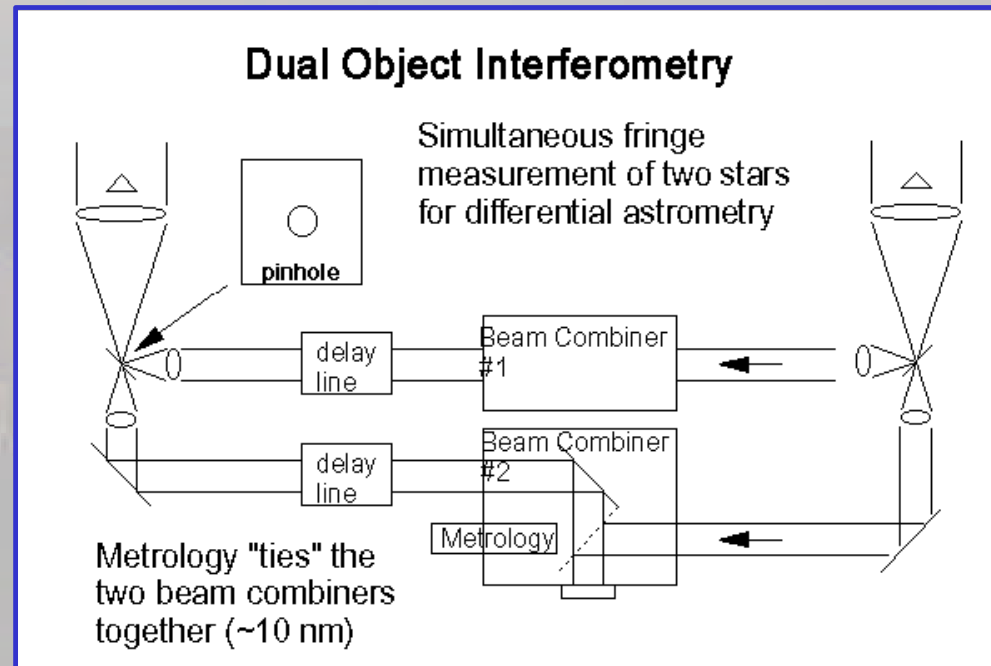
# Phase tracking

The “easy” way:

- Use a broad-band fringe tracking channel and lock onto white-light fringe.
- Follow the fringe motion in real-time and sample fast enough so that fringe motion between samples is  $\ll 180$  degrees.
- Can use a broad-band channel to **phase-reference** other narrow-band channels:
  - Increases effective coherence time to **seconds**.
  - Equivalent to self-referenced adaptive optics on the scale of the array.
- Because it's a high precision technique it has  $\sim 2.5$  mag poorer sensitivity than group-delay tracking.

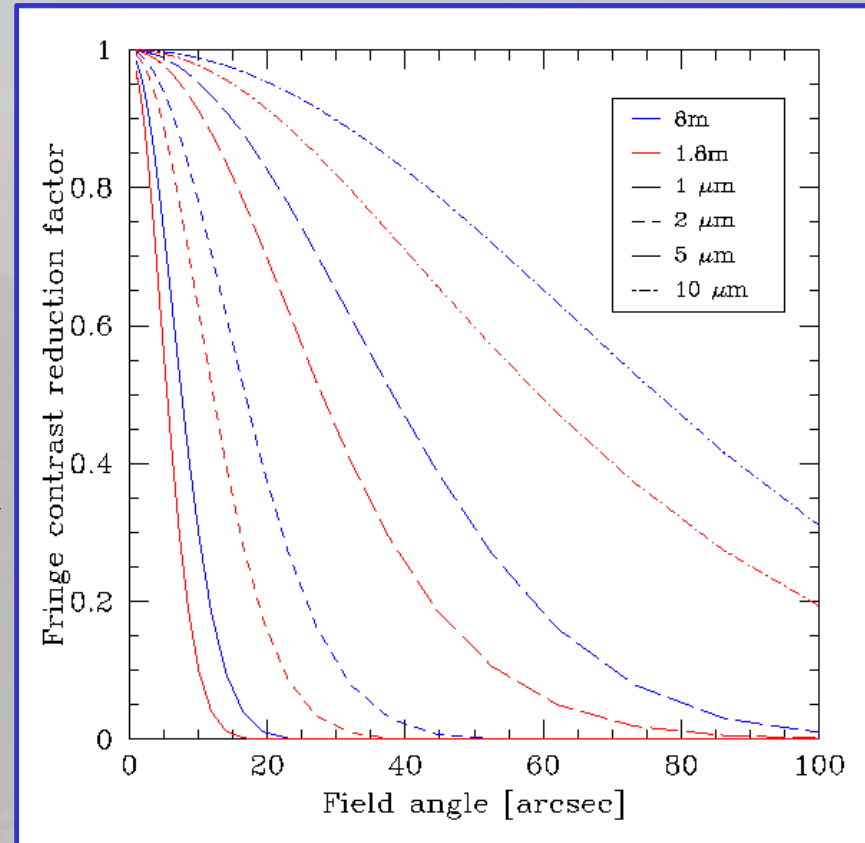
# Off-axis phase referencing

- The “difficult” way: **dual-feed** operation. This is what **PRIMA** aims to deliver:
  - Use bright **off-axis** reference star to monitor the atmospheric perturbations in real-time.
  - Feed corrections to **parallel** delay-lines observing science target.
  - Use a **metrology** system to tie two optical paths together.
  - In principle can extend **effective** coherence time by orders of magnitude if the white-light fringe is tracked.



## Dual-feed interferometry (cont'd)

- Practical issues:
  - Off-axis wavefront perturbations become uncorrelated as field angle increases and  $\lambda$  decreases.
  - With 1' field-of-view <1% of sky has a suitably bright reference source (H<12).
  - Metrology is non-trivial.
  - Laser guide stars are not suitable reference sources.



Off-axis reduction in mean visibility for the VLTI site as a function of D and  $\lambda$ .

# Good observables

- In the absence of a PRIMA-like system, optical/IR interferometrists have had to rely upon measuring phase-type quantities that are immune to atmospheric fluctuations.
- These are self-referenced methods - i.e. they use simultaneous measurements of the source itself:
  - Reference the phase to that measured at a different wavelength - **differential phase**:
    - Depends upon knowing the source structure at some wavelength.
    - Need to know atmospheric path and dispersion.
  - Reference the phase to those on different baselines - **closure phase**:
    - Independent of source morphology.
    - Need to measure many baselines at once.

## Closure phases (i)

- For an array of  $N$  telescopes, with  $N-1$  unknown phase perturbations we can measure  $N(N-1)/2$  visibility phases.
- This implies that there must be  $(N-1)(N-2)/2$  quantities we can infer from our measurements that only depend on the source structure.
- The corresponding closure phases are one such set of these.

$N_{\text{tels}}$	3	4	5	8	$N$
$N_{\text{bas}}$	3	6	10	28	$N(N-1)/2$
$N_{\text{clos\_indep}}$	1	3	6	21	$(N-1)(N-2)/2$
$N_{\text{clos\_all}}$	1	4	10	56	$N(N-1)(N-2)/6$
$\text{Frac}_{\text{phase}}$	0.33	0.50	0.60	0.75	$1-(2/N)$



## Closure phases (ii)

- Measure visibility phase ( $\Phi$ ) on three baselines **simultaneously**.
- Each is perturbed from the true phase ( $\phi$ ) in a particular manner:

$$\Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2$$

$$\Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3$$

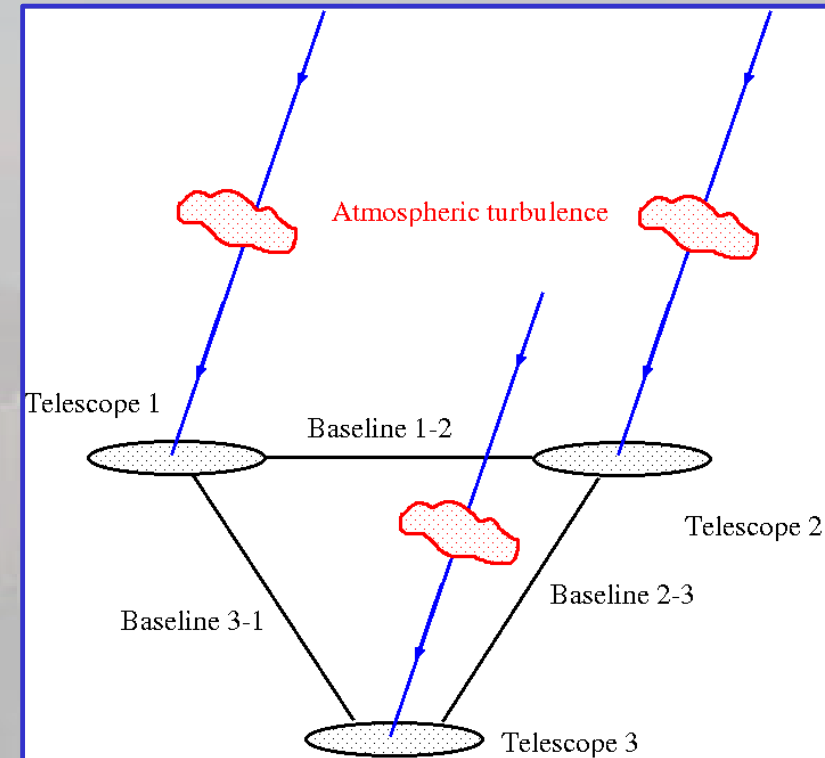
$$\Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1$$

- Construct the **linear combination** of these:

$$\Phi_{12} + \Phi_{23} + \Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$

The error terms are antenna dependent

The source information is baseline dependent.



## Using “good” observables

- **Average** them (properly) over many realisations of the atmosphere.
- Differential phase, **if** we are comparing with the phase at a wavelength at which the source is unresolved, is a **direct proxy** for the Fourier phase we need.
  - Can then Fourier invert straightforwardly.
- Closure phase is a peculiar linear combination of the true Fourier phases:
  - In fact, it is the argument of the product of the visibilities on the baselines in question, hence the name **triple product (or bispectrum)**.

$$V_{12}V_{23}V_{31} = |V_{12}| |V_{23}| |V_{31}| \exp(i[\Phi_{12} + \Phi_{23} + \Phi_{31}]) = T_{123}$$

- So we have to use the closure phases as additional constraints in some nonlinear iterative inversion scheme.

# Outline

- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - Beam combination
  - Spatial wavefront fluctuations
  - Temporal wavefront fluctuations
  - **Sensitivity**
  - Calibration

## Sensitivity (i)

- We have mentioned earlier that sensitivity in an interferometric context really means two things:
  - It must be possible to **stabilize** the array in real time against atmospheric-induced fluctuations of the OPD.
  - Once this is satisfied, we need to be able to **build up** enough signal-to-noise on the astronomical fringe parameters of interest.
- The essential implication of this is that the “instantaneous” fringe detection S/N has to be high enough to “track” fringes.
- This signal/noise ratio basically scales as:

$$S/N \propto [VN]^2 / [(N+N_{\text{dark}})^2 + 2(N+N_{\text{dark}})N^2V^2 + 2(N_{\text{pix}})^2(\sigma_{\text{read}})^4]^{-1/2}$$

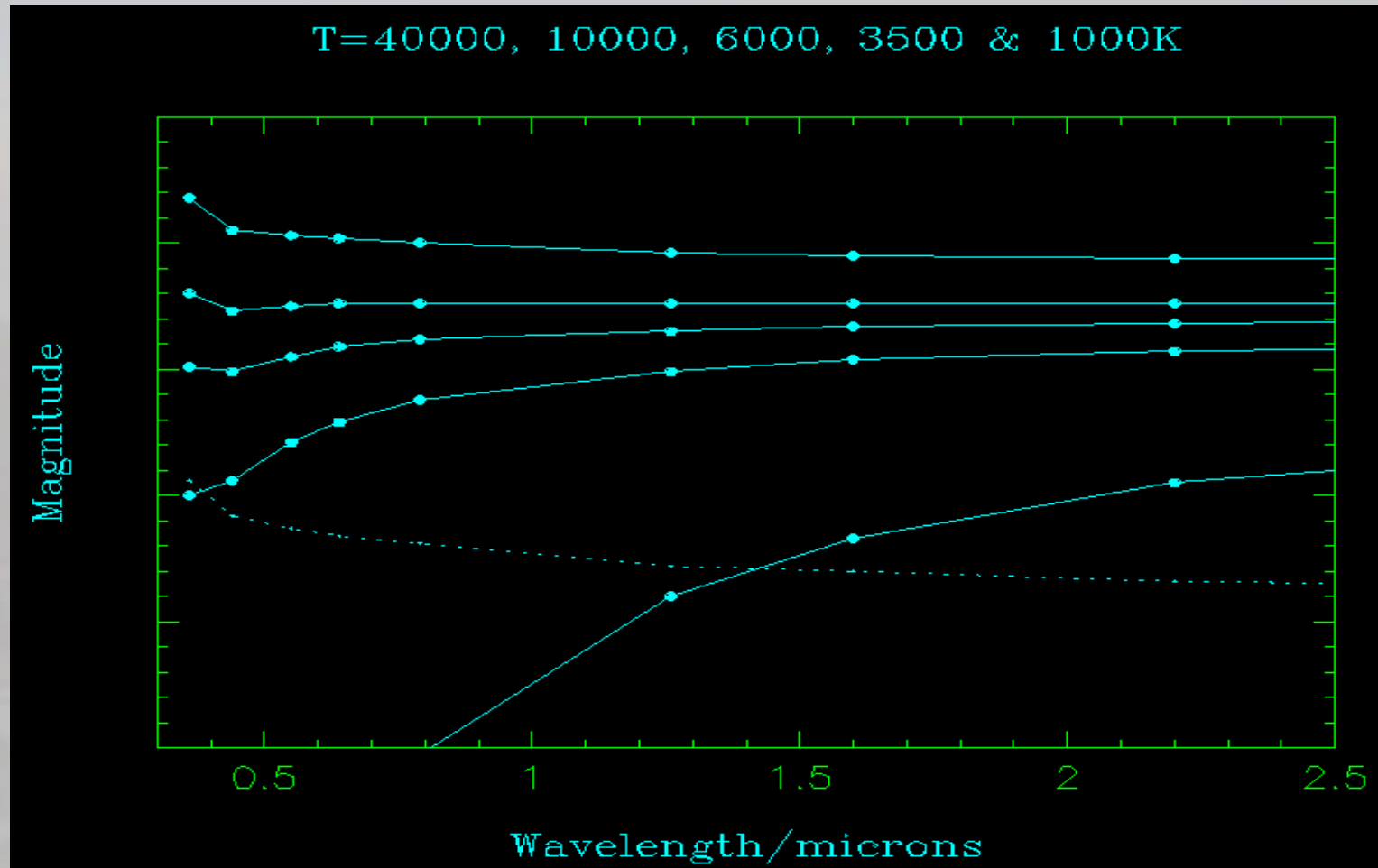
with  $V$  = apparent visibility,  $N$  = detected photons,  $N_{\text{dark}}$  = dark current,  $N_{\text{pix}}$  = number of pixels,  $\sigma_{\text{read}}$  = readout noise/pixel.

## Sensitivity (ii)

$$\begin{aligned} S/N &\sim [VN]^2/[N^2 + 2V^2N^3 + 2N_p^2\sigma^4]^{1/2} \\ &\sim [V^2N]^\alpha, \text{ with } \alpha = 1/2 \text{ or } 1. \end{aligned}$$

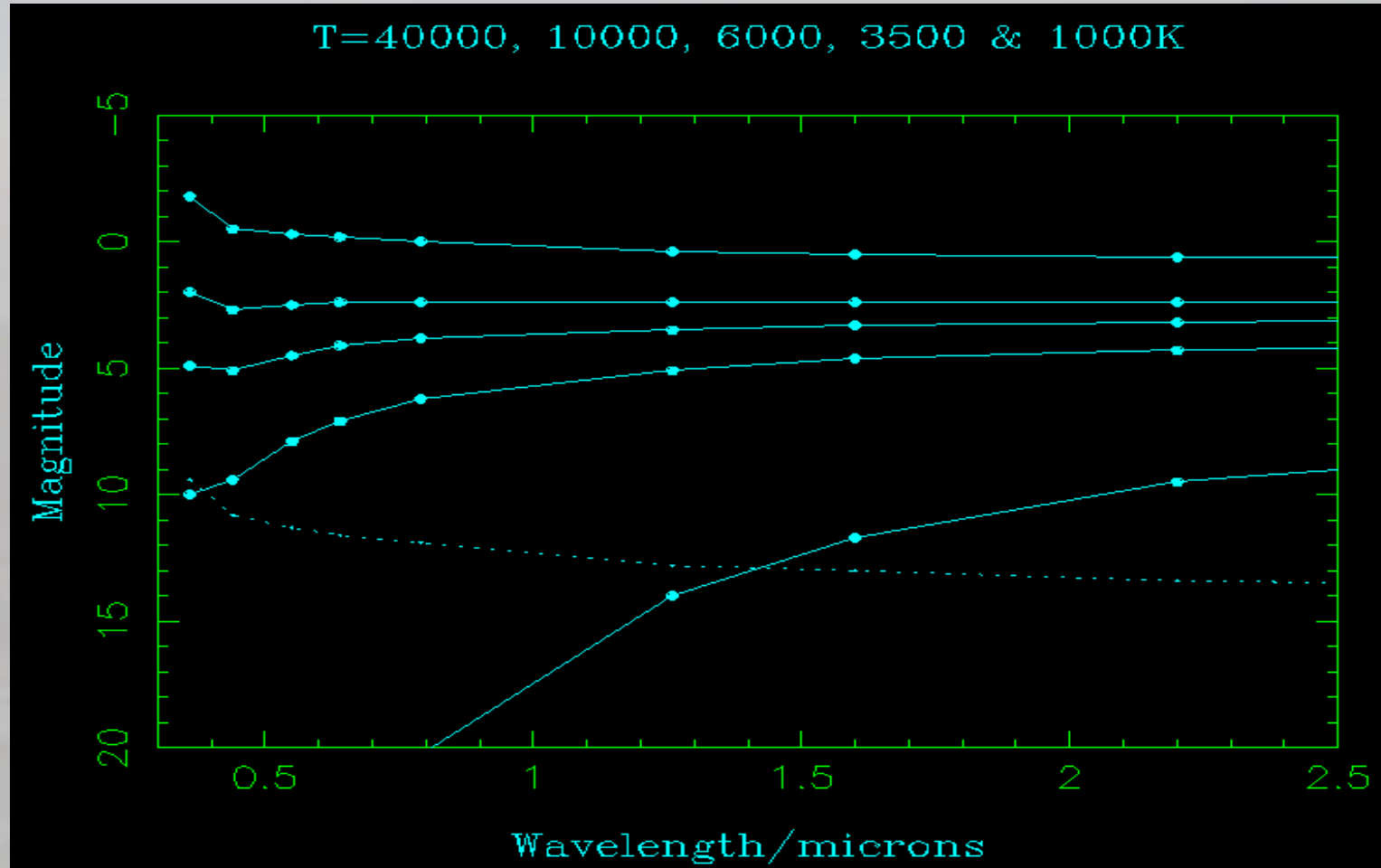
- In general we want this to be  $> 1$ .
  - Good fringe visibility is more important than more light.
  - Resolved sources have  $V \ll 1$ . This implies very large reductions in the sensitivity of an interferometric array if the source being used to stabilize the array is resolved.
  - On the longest interferometric baselines, the S/N will always be low.
  - Bright sources are generally big - the small ones are faint!

# Sensitivity



Apparent magnitudes of 1mas blackbodies of different temperatures.

# Sensitivity



Apparent magnitudes of 1mas blackbodies of different temperatures.

## Sensitivity (iii)

- In summary:
  - Need to have enough  $V^2N$  to stabilize the array.
  - Then we need to have enough integration time to build up a useful S/N on the science signal.
  - The problem is that many sources of interest will have small  $V$ .
- Solutions:
  - Use off-axis reference sources for stabilization (PRIMA).
  - Decompose all long baselines into shorter ones where  $V$  is not so low.



# Outline

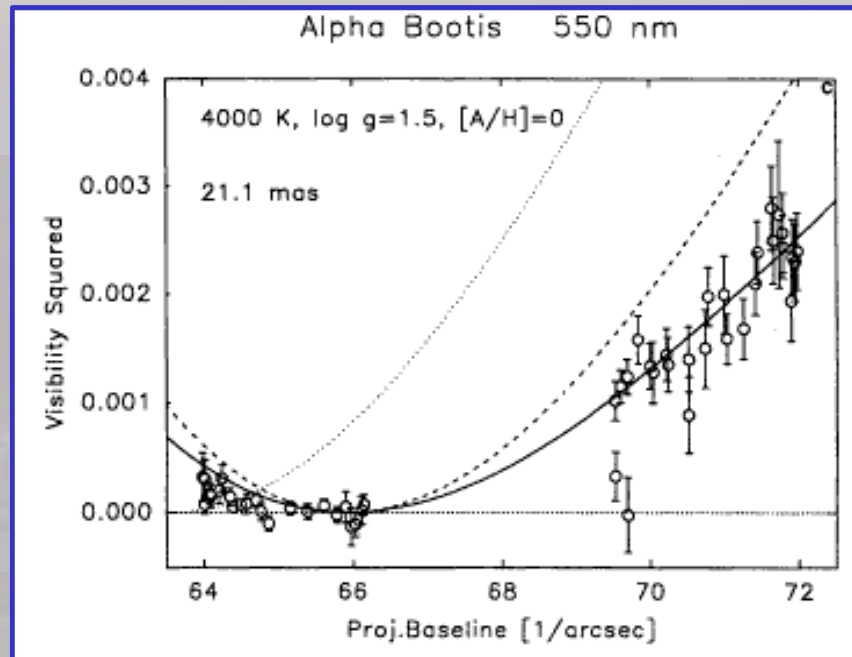
- What are the things that make interferometry less than straightforward in practice?
  - Sampling of the  $(u, v)$  plane
  - Delay lines
  - Beam combination
  - Spatial wavefront fluctuations
  - Temporal wavefront fluctuations
  - Sensitivity
  - Calibration

# Calibration

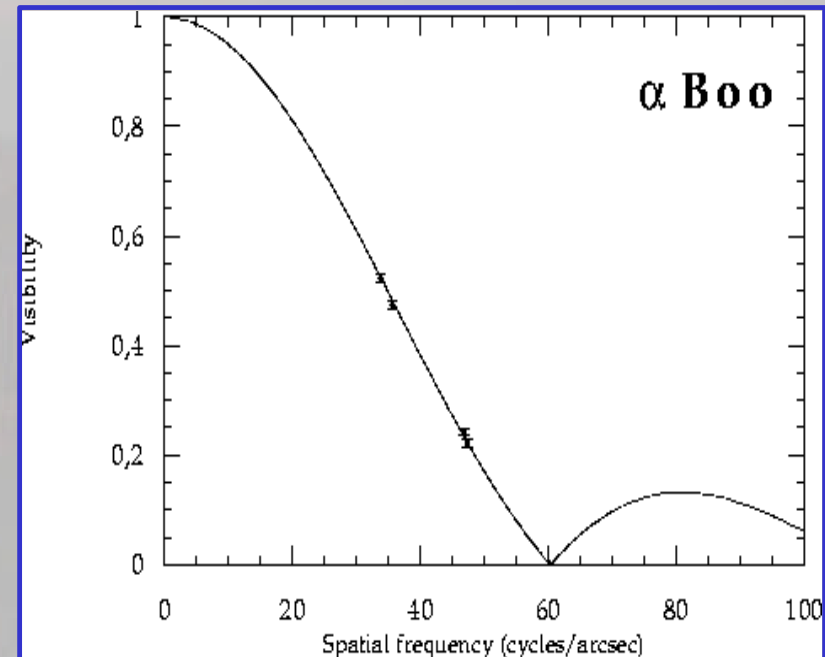
- The basic observables we wish to estimate are **fringe amplitudes** and **phases**.
- In practice the **reliability** of these measurements is generally limited by systematic errors, not the S/N we have just discussed.
- Hence there is a crucial need to **calibrate** the interferometric response:
  - Measurements of sources with known amplitudes and phases:
    - Unresolved targets close in time and space to the source of interest.
  - Careful design of instruments:
    - Spatial filtering.
  - Measurement of quantities that are less easily modified by systematic errors:
    - Phase-type quantities.

# Examples of real data

Measurements with the NPOI



Measurements with FLUOR



Perrin et al, AA, 331 (1998)

So you need to know what is required for the science.

# Summary

- Sampling of the  $(u, v)$  plane
  - What is needed for the scientific questions being addressed.
  - Will the array operate satisfactorily on these baselines.
- Delay lines
  - Intrinsic performance, dispersion at long baselines.
- Spatial fluctuations
  - Impact on sensitivity, potential limitations of AO.
- Temporal fluctuations
  - Impact on sensitivity, need for fringe tracking.
  - Good observables and how these are used.
- Sensitivity
  - An appropriate measure of this in terms of stabilizing the array.
  - $V^2N$  scaling.
- Calibration
  - Importance of this to deliver useful science.

