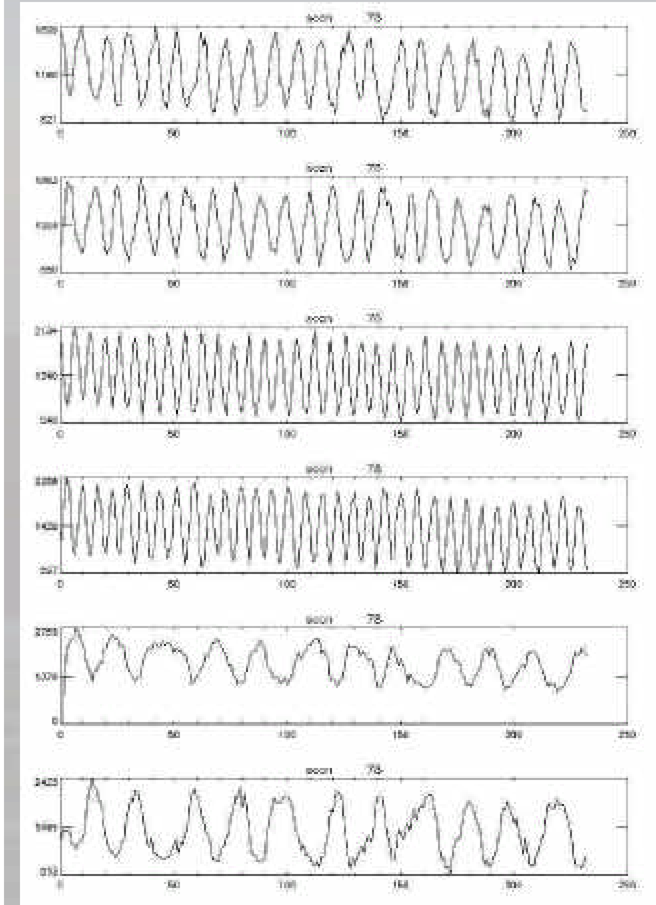
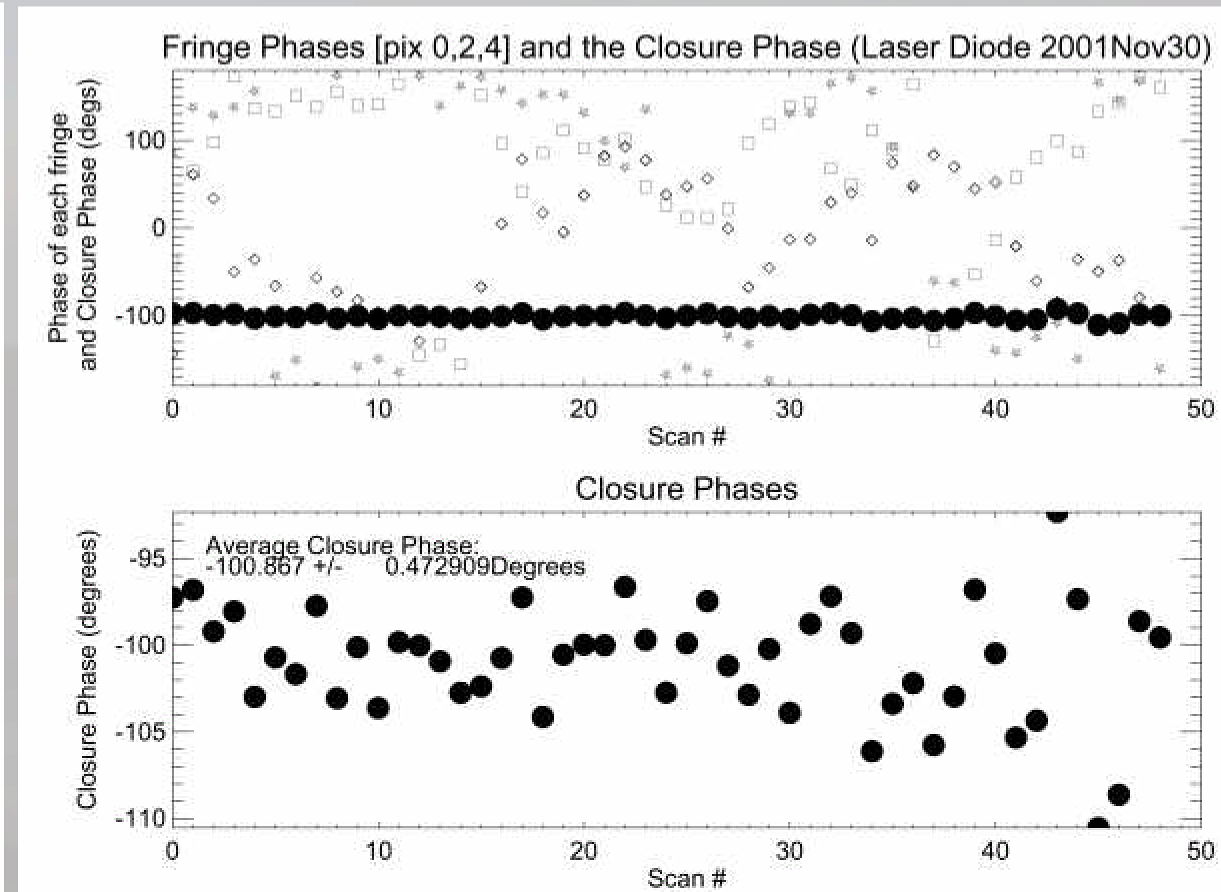
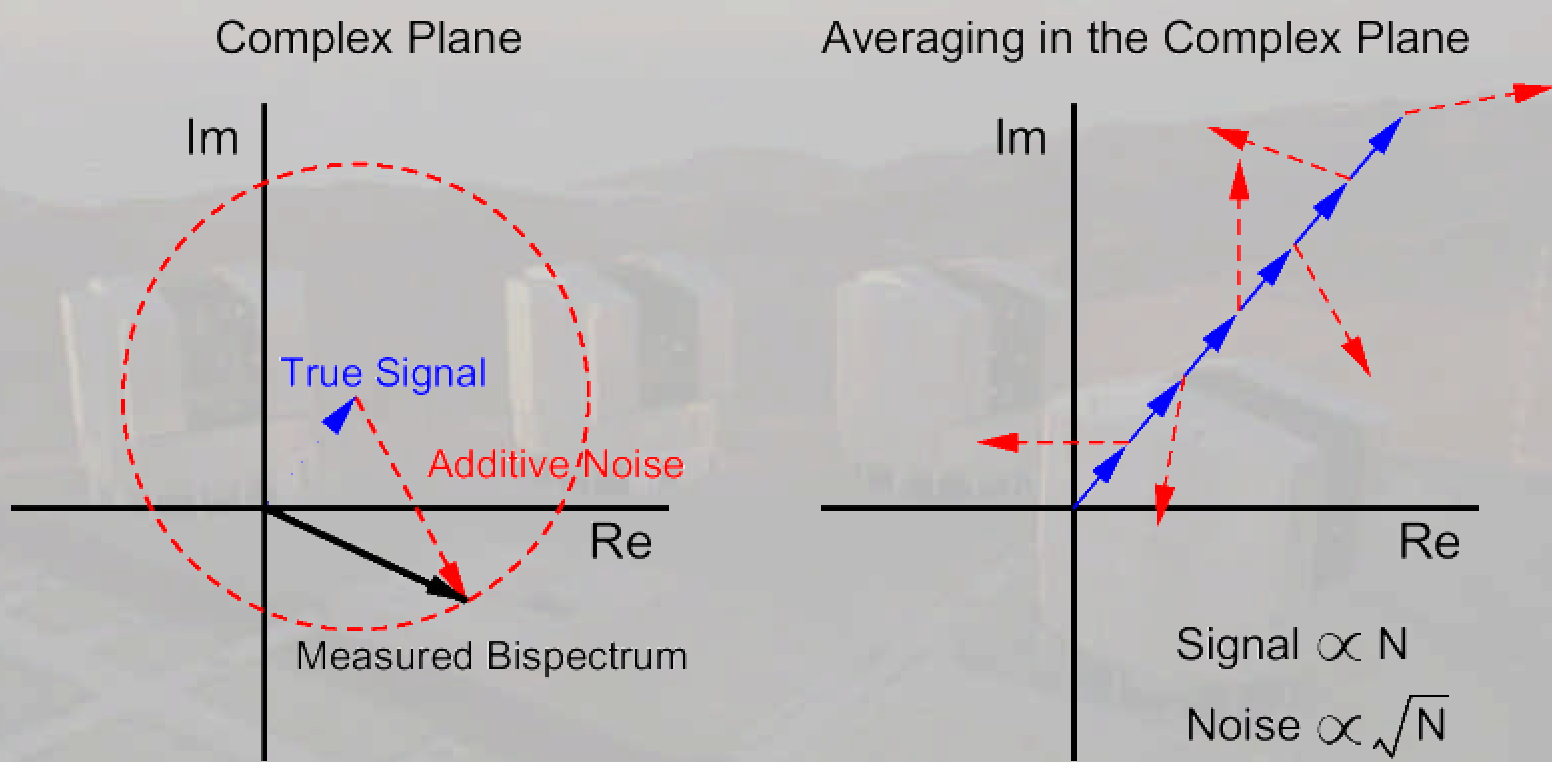
Closure Phase is a Good Observable

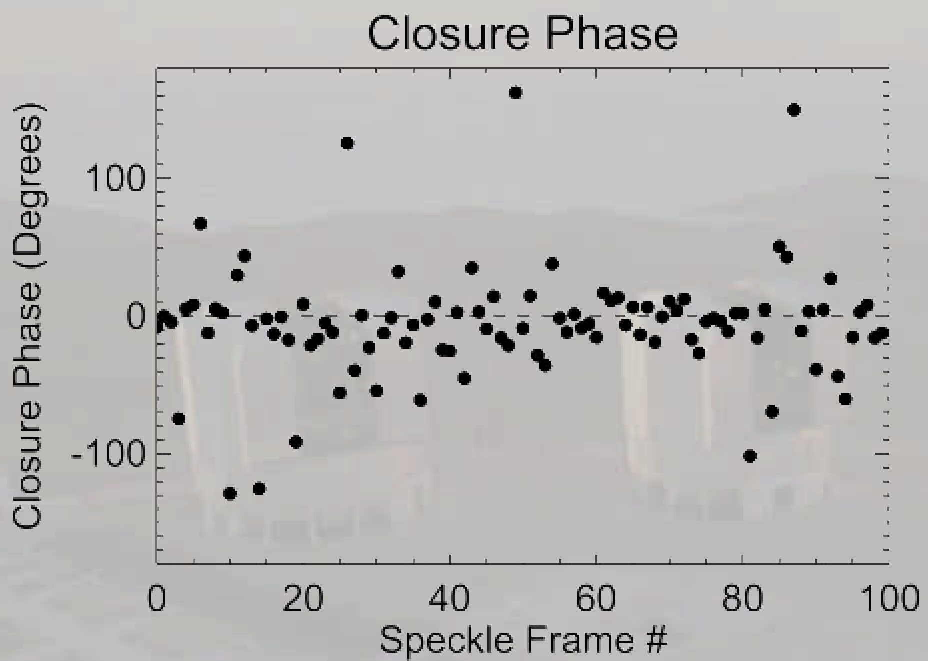


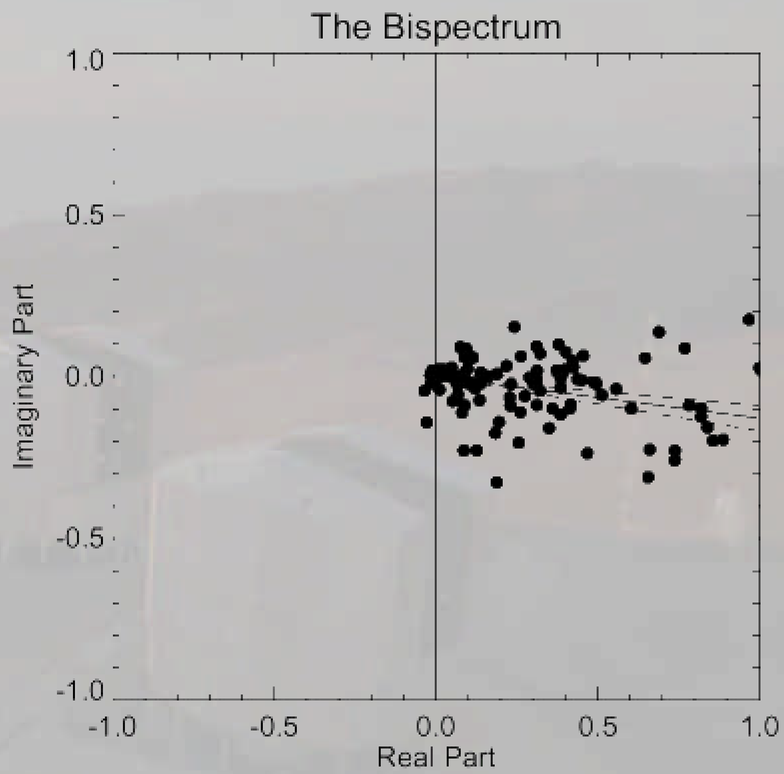


Closure Phase Averaging



Bispectrum in the Complex Plane





How Much Phase Information?

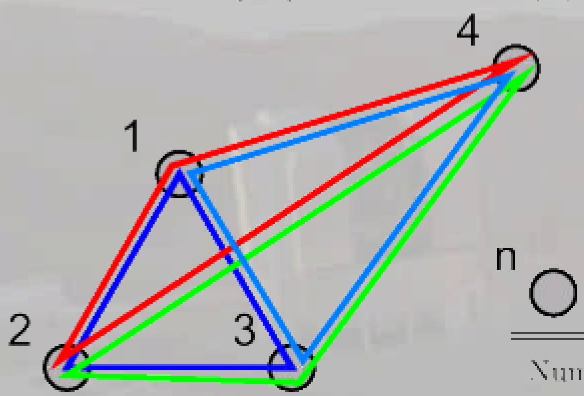
Closure Phases are not all independent from each other.

Number of Closure Phases

$$\binom{N}{3} = \frac{(N)(N-1)(N-2)}{(3)(2)},$$

Number of Fourier Phases

$$\binom{N}{2} = \frac{(N)(N-1)}{2}$$



Number of Independent Closure Phases

$$\binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$$

Number of Telescopes	Number of Fourier Phases	Number of Closing Triangles	Number of Independent Closure Phases	Percentage of Phase Information
3	3	1	1	33%
7	21	35	15	71%
21	210	1330	190	90%
27	351	2925	325	93%
50	1225	19600	1176	96%

Closure Amplitudes too

$$
\begin{aligned}
A_{ijkl} &= \frac{|\tilde{\mathcal{V}}_{ij}^{\text{measured}}||\tilde{\mathcal{V}}_{kl}^{\text{measured}}|}{|\tilde{\mathcal{V}}_{ik}^{\text{measured}}||\tilde{\mathcal{V}}_{jl}^{\text{measured}}|} \\
&= \frac{|\tilde{G}_i||\tilde{G}_j||\tilde{\mathcal{V}}_{ij}^{\text{true}}||\tilde{G}_k||\tilde{G}_l||\tilde{\mathcal{V}}_{kl}^{\text{true}}|}{|\tilde{G}_i||\tilde{G}_k||\tilde{\mathcal{V}}_{ik}^{\text{true}}||\tilde{G}_j||\tilde{G}_l||\tilde{\mathcal{V}}_{jl}^{\text{true}}|} \\
&= \frac{|\tilde{\mathcal{V}}_{ij}^{\text{true}}||\tilde{\mathcal{V}}_{kl}^{\text{true}}|}{|\tilde{\mathcal{V}}_{ik}^{\text{true}}||\tilde{\mathcal{V}}_{jl}^{\text{true}}|}.
\end{aligned}
$$

Closure amplitudes have not been used effectively in optical interferometry because fringe amplitude fluctuations are mostly caused by variable atmospheric coherence (and because there are few 4-telescope arrays).

However, closure amplitudes should be useful for interferometers using spatial filters such as single-mode fibers.

Important Properties of Closure Phases

More robust to calibration error

- Insensitive to **telescope-based** errors (e.g., atmospheric dispersion)
- Atmospheric turbulence generally does not bias measurement (unlike Visibility^2)
- SNR is not as bad as some might suggest! (more later)
- Reasonable hope of measurement error reducing as root (N), until …?
- But there are some potentially troubling biases (e.g., photon noise, subtle readout noise correlations, chromatic effects for broadbands)

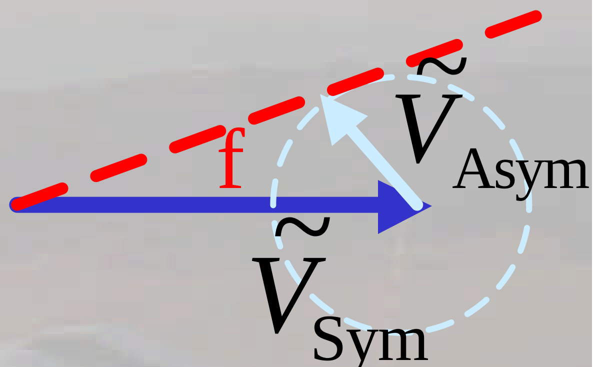
Sensitive to asymmetries in brightness distribution

- Bispectrum REAL for point-symmetry (Φ_{CP} = 0 or 180 degs)
- Critical for validating Vis^2 modeling
- Necessary for imaging (if no phase referencing)

Estimate the Magnitude of Closure Phase

It is straightforward to obtain an order-of-magnitude estimate for the "typical" closure phase for a known object distribution:

$$\left|\text{Closure Phase (radians)}\right| \approx \frac{\text{Asymmetric Flux}}{\text{Symmetric Flux}}$$



The amount of "Asymmetric" Flux should be based on the resolution of the baselines (Nothing is asymmetric if its unresolved!)

Example: For an unequal binary system, a closure phase measurement (radians) will typically be roughly the brightness ratio if the binary separation is resolved.

Closure Phase Example: Binaries