From visibilities to science with simple models

EuroWinter School

Observing with the Very Large Telescope Interferometer

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Why measure limb-darkening?

Some of the reasons:

- Must know it to measure effective temperature
- Interpret eclipsing binary light curves
- Diagnostic for stellar models
- Modelling complex brightness distributions from interferometric data

Measurement Techniques

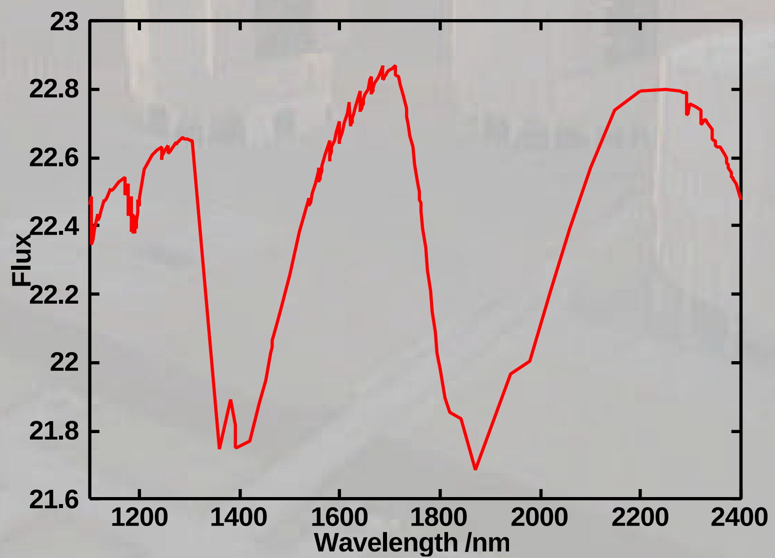
- Interferometry!
- Lunar occultations
- Gravitational microlensing
- Light curves - stars in eclipsing multiple systems

Example Proposal

- Aim: compare continuum limb-darkening of Mira variables with atmospheric models
- Use interferometry with VLTI
- Requirements (for *all* limb-darkening studies):
  - precise measurement of visibility
  - angular resolution
  - spectral resolution
  - sensitivity
  - multiple baselines

Mira-specific requirements

- Need measurements at specific pulsation phases — constrains array reconfigurations
- May be better diagnostics for models than limb-darkening!
- Choose bandpass carefully:



Models in Fourier Space

Interferometers measure *visibilities*, related to sky brightness distribution by a Fourier Transform:

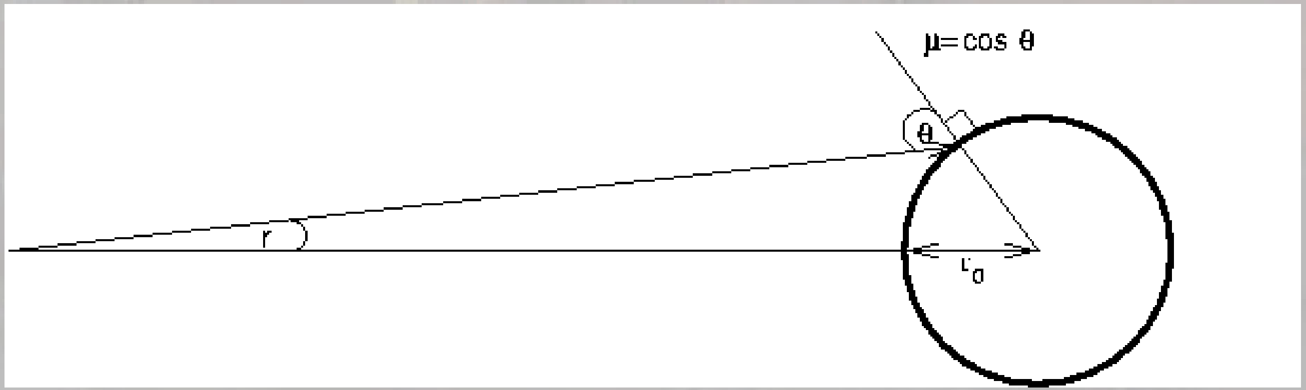
$$V(u,v) = \iint dx\, dy\; I(x,y)\, e^{-2pi(ux+vy)}$$

For a circularly-symmetric disk with centre-to-limb intensity profile *I*(*r*) this becomes a Hankel Transform:

$$V(d_l) = 2p \int I(r)\, J_0(d_l r)\; r\, dr$$

Empirical limb-darkening models

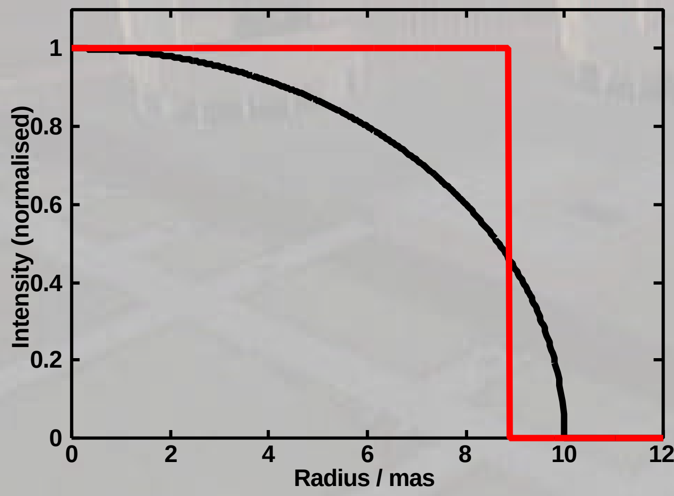
- Simple functions
- Physically reasonable
- Typically expansions in $m(r) = \sqrt{1 - \frac{r^2}{r_0^2}}$:



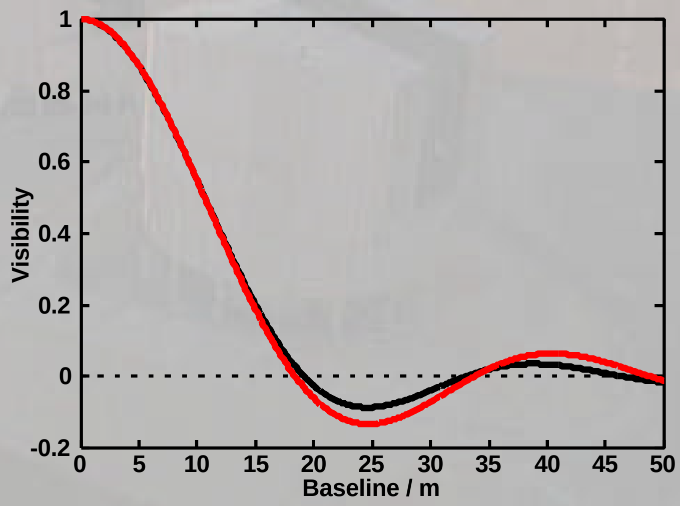
Empirical models in Fourier Space

- Examples:
  - Taylor expansion: $$\frac{I(r)}{I_0} = 1 - \sum_{n=1}^{n_{max}} a_n (1 - m(r))^n \quad r < r_0$$ $$I(r) = 0 \quad r > r_0$$
  - Hestroffer model: $$\frac{I(r)}{I_0} = m^a \quad r < r_0$$
- Analytical expressions for their Hankel transforms can be derived:

Real space



Fourier space



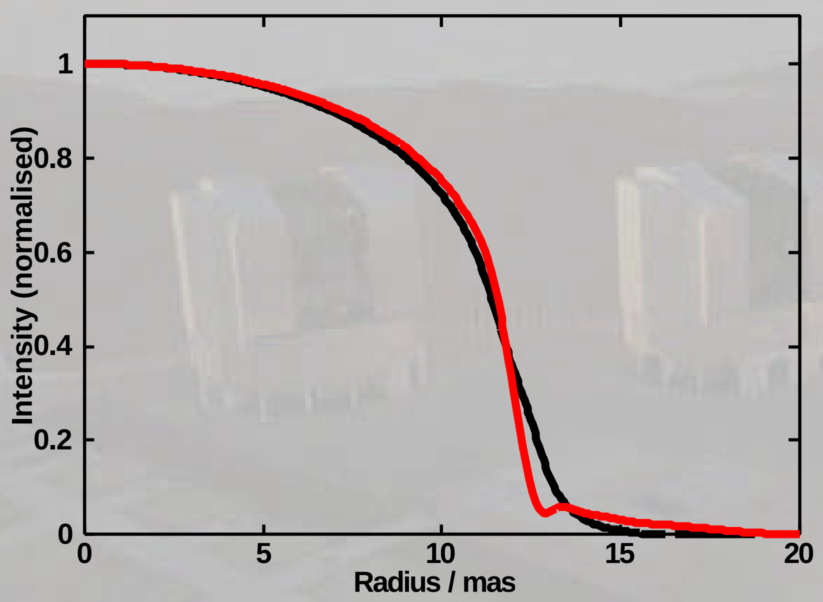
Realistic models

- Models from Hofmann, Scholz & Wood (1998) A&A 339, 846 (HSW98)
- Dynamical models with realistic atmospheres
- Parameters similar to ο Ceti and R Cassiopeiae
- Both fundamental-mode and first-overtone pulsators
- Models predict centre-to-limb variation of intensity as a function of wavelength

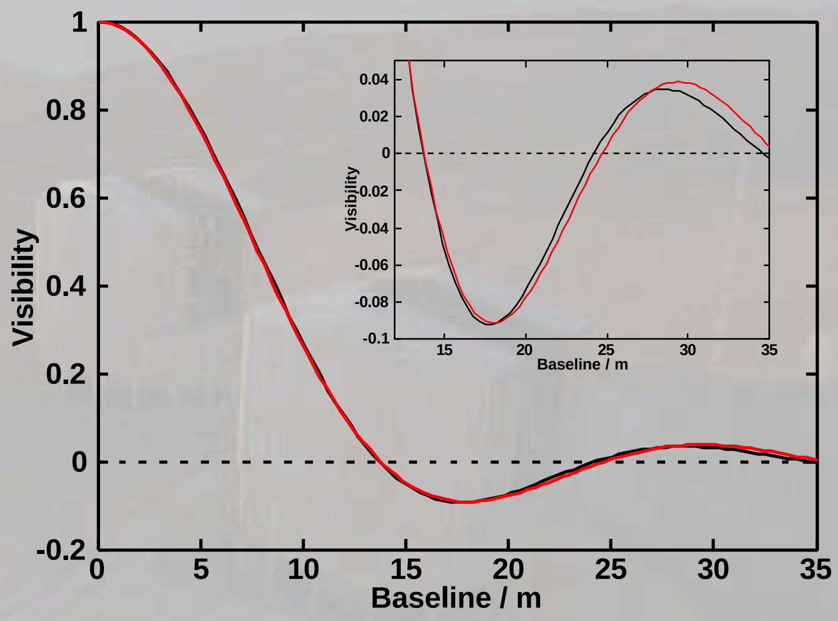
Realistic models: limb-darkening

HSW98 M10 v E8380

Real space



Fourier space



Limb-darkening: Points to note

- Larger, more limb-darkened disk gives *identical* short-baseline visibilities to smaller, less limb-darkened disk
- Limb-darkening suppresses second lobe
- Miras are an extreme case; better agreement between different models of less extended stars
- Taylor expansion has many problems

Example Proposal: Requirements

- Requirements for nearest ~5 southern Miras:
  - precise measurement of visibility: *V*=0.02 to +/-5%
  - Diameter 20 mas: need 25m max. baseline for 1.3μm measurement
  - spectral resolution: ~30
  - sensitivity: *J* ~0
  - multiple baselines: at least 2
  - closure phase useful (may be no null)

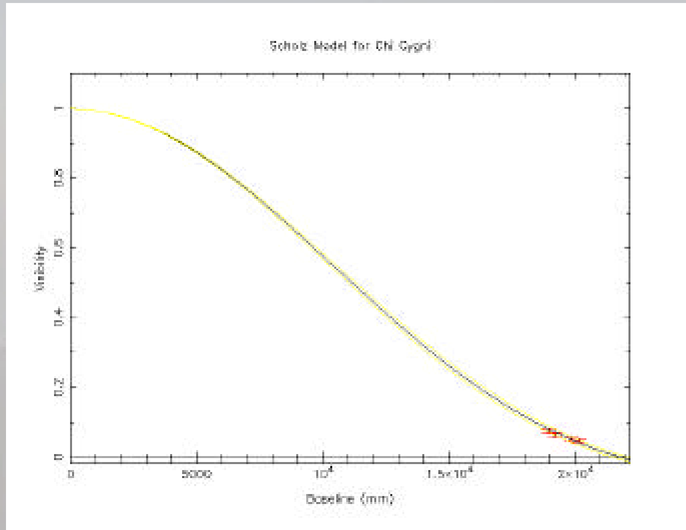
Example data analysis

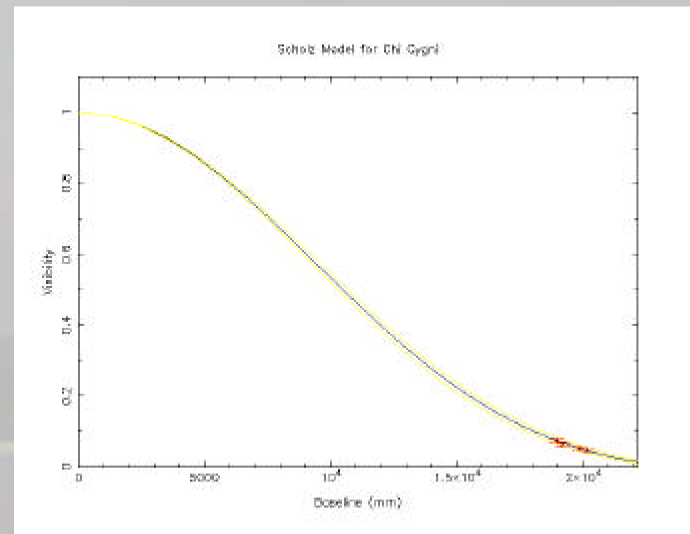
- Present real COAST data:
  - χ Cygni near minimum light
  - Broad-band 1.3 micron data
  - 6 baselines
  - Closure phases
- Compare data with models appropriate to this pulsation phase:
  - HSW98 M05 v P05
  - More extreme limb-darkening
  - Models can be distinguished at lower resolution
- Will show sub-sets of data for illustration

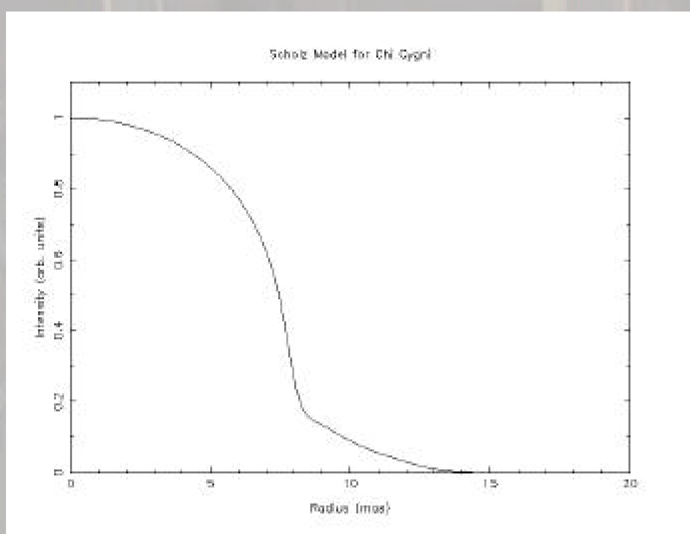
Model-fitting

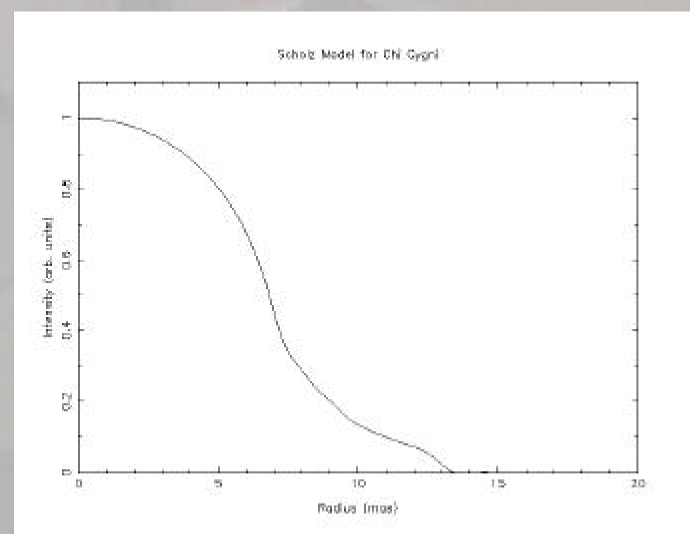
- Each model predicts a *Centre-to-Limb Variation* of intensity (CLV)
- Calculate Hankel transform numerically, for assumed disk size
- Fit this model to the data by scaling model baselines — equivalent to scaling size of model star
- Compare χ^2 values for different models

One baseline (silly!)



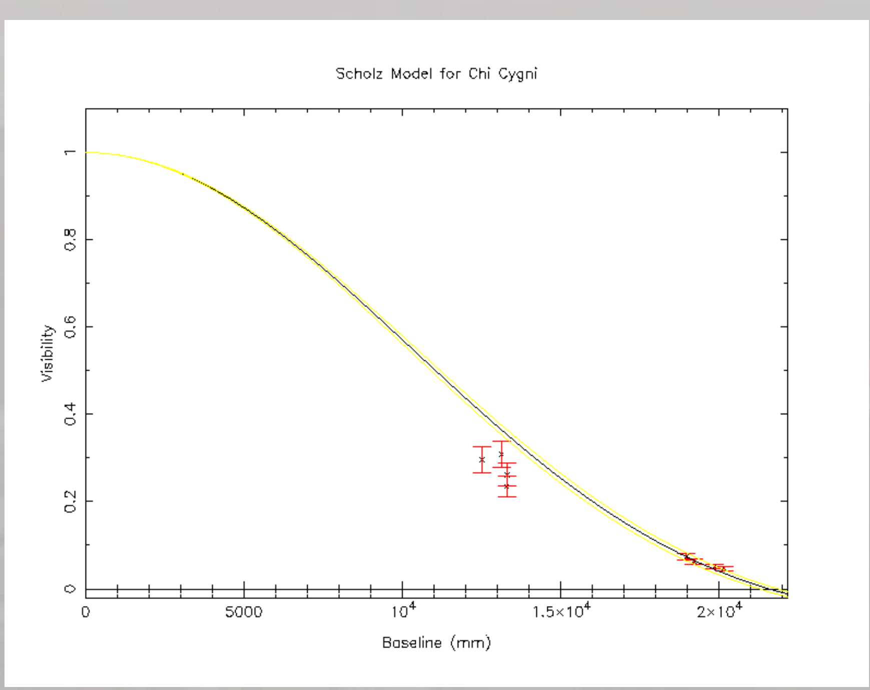


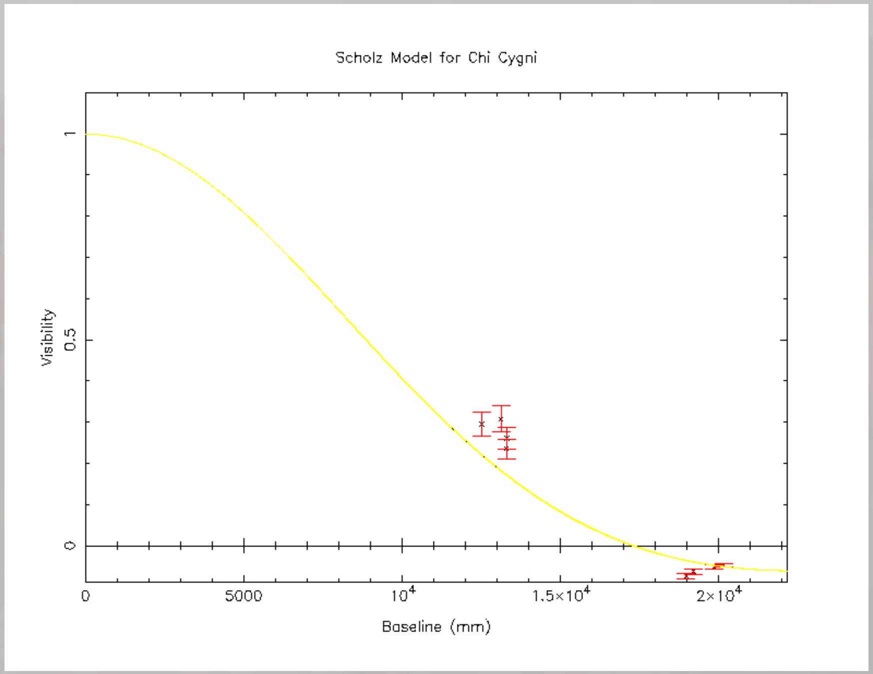




Two baselines - M05 model

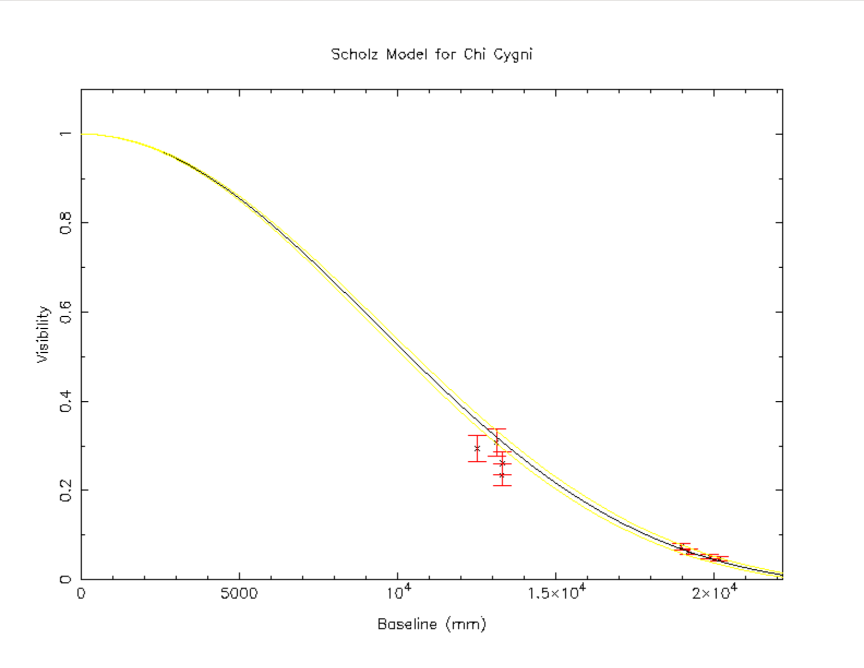
Two solutions, both poor fits



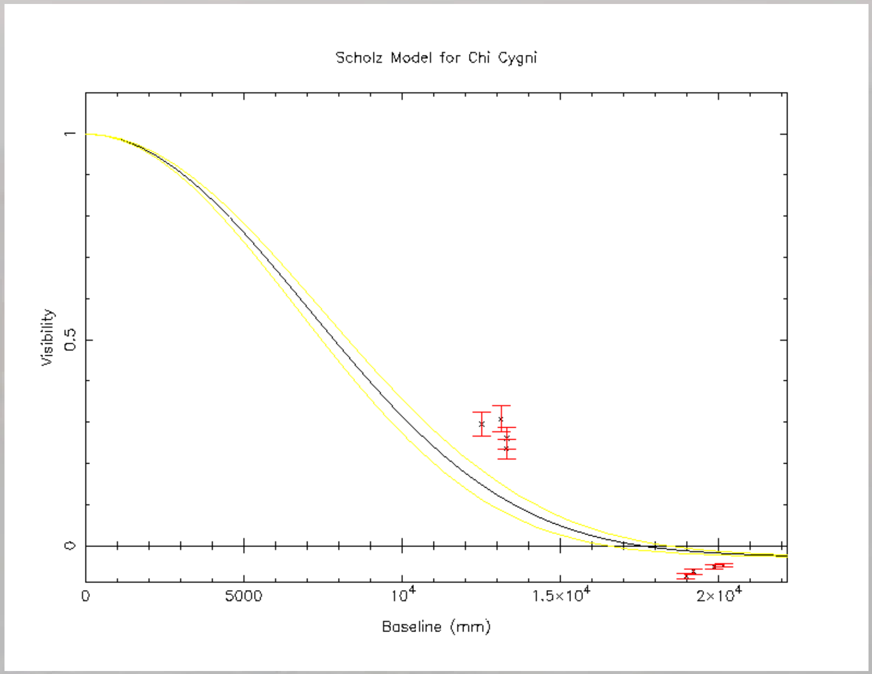


Two baselines - P05 model

Reasonable fit

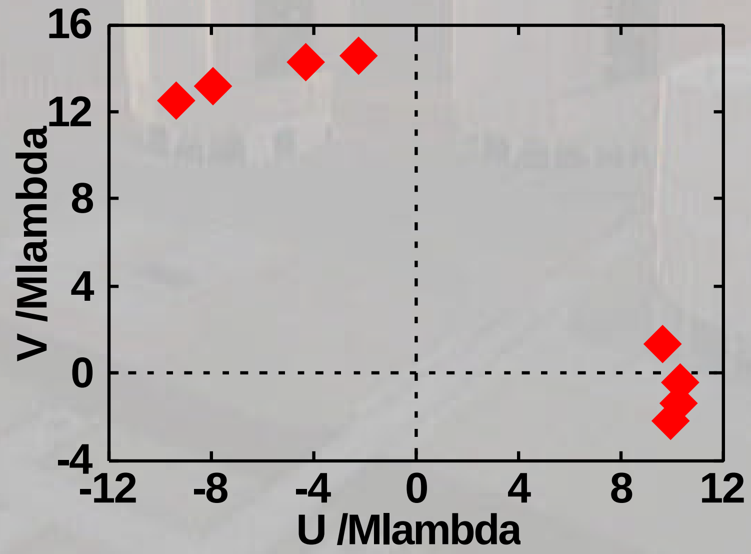


Poor fit

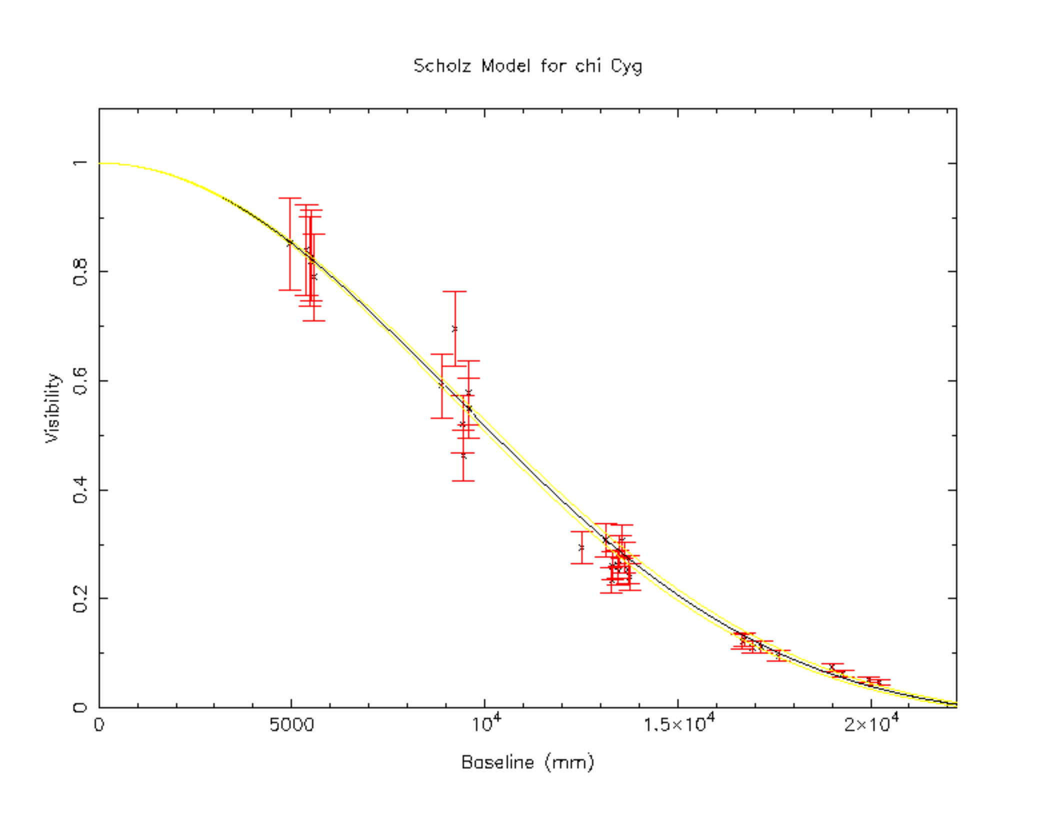


Two baselines - observing time with VLTI

- Assume three ATs present ⇒ **no reconfiguration**
- Use AMBER in single-baseline mode
  - Switch between two of three available baselines every ~15 min
- Visibility curves shown have hour angle range 2h 30m, but could secure four visibilities on each baseline in ~2h



Six baselines - P05 model



Conclusions

- Limb-darkening measurements feasible with VLTI + first generation instruments:
  - precise measurement of visibility
  - angular resolution
  - spectral resolution
  - sensitivity
  - multiple baselines
- Care needed when analysing data
- Spatial constraints from interferometry may show up deficiencies in models